Published in:
"Generation of Low Temperature and It's Applications," edited by T. Ohtsuka and Y. Ishizaki, Shonan Tech. Center, Kamakura, Japan, (2003), pp. 1-20.

Thermodynamics of Regenerative Refrigerators

Ray Radebaugh
National Institute of Standards and Technology
Boulder, Colorado 80305 USA

Thermodynamic fundamentals of closed systems

Figure 1 shows a schematic of a refrigerator and the important thermodynamic quantities associated with it. The function of the refrigerator is to absorb the heat flow $\dot{Q}_c$ from some cold reservoir at a temperature T_c and reject the heat flow $\dot{Q}_0$ to the surrounding at some ambient temperature T_0. The net input power required to operate the refrigerator is $\dot{W}_{co}$ and the refrigerator may provide some external power flow $\dot{W}_{exp}$ from an expander. Many refrigeration systems either produce no expansion work, or they recover the expansion work internally, in which case there still is no expansion work crossing the system boundary. Thus, $\dot{W}_{exp}$ is often zero for a complete refrigeration system. The coefficient of performance *COP* of a refrigerator is defined as

$$COP = \frac{\dot{Q}_c}{\dot{W}_{co}}. \tag{1}$$

A relationship between the parameters $\dot{Q}_c, T_c, \dot{Q}_0, T_0, \dot{W}_{co}, \dot{W}_{exp}$, and *COP* may be found by utilizing the first and second laws of thermodynamics. For a **closed system** in which no mass crosses the system boundaries the first law (conservation of energy) is given as

$$\textbf{First Law}: \quad \dot{W}_{co} - \dot{W}_{exp} = \dot{Q}_0 - \dot{Q}_c + \frac{d(mu)}{dt}, \tag{2}$$

where $d(mu)/dt$ is the time rate of change of the internal energy of the refrigerator of mass m and specific internal energy u. For steady-state conditions $d(mu)/dt = 0$. The second law (balance of entropy) is given as

$$\textbf{Second Law}: \quad \frac{\dot{Q}_c}{T_c} - \frac{\dot{Q}_0}{T_0} + \dot{S}_{irr} = \frac{d(ms)}{dt}, \tag{3}$$

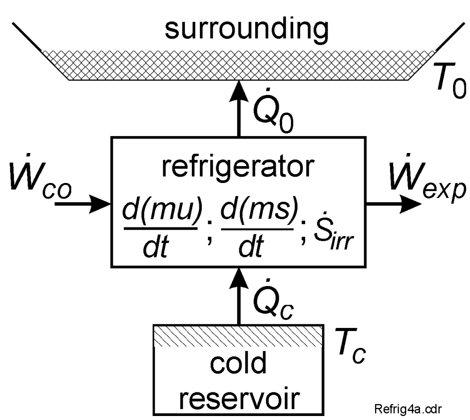

Figure 1. Schematic of a refrigerator shown as a closed thermodynamic element along with the important thermodynamic parameters.

where T_c is the cold end temperature, T_0 is the ambient temperature, $d(ms)/dt$ is the time rate of change of the entropy of the refrigerator and $\dot{S}_{irr}$ is the rate of entropy generation due to irreversible processes, which is always positive. For steady-state conditions $d(ms)/dt = 0$. We see that the second law relates the heat flows at the warm and cold ends of the refrigerator to the temperatures.

By solving Eq. (2) for $\dot{Q}_0$ and substituting into Eq. (3) we combine the first and second laws and obtain the following expression for the *COP* under steady-state conditions:

$$COP \equiv \frac{\dot{Q}_c}{\dot{W}_{co}} = \frac{T_c}{T_0 - T_c}\left[1 - \frac{(T_0 \dot{S}_{irr} + \dot{W}_{exp})}{\dot{W}_{co}}\right]. \tag{4}$$

As this equation shows, if the expansion work leaves the system and is not recovered internally, it represents lost work and degrades the *COP* of the refrigerator. Similarly, the irreversible entropy production $\dot{S}_{irr}$ multiplied by the ambient temperature represents another form of lost work in the system. A common parameter used to express the performance of cryocoolers is the reciprocal of the *COP*, known as the specific power,

$$p_s \equiv \frac{1}{COP} = \frac{\dot{W}_{co}}{\dot{Q}_c}. \tag{5}$$

For an ideal refrigerator there are no irreversible processes ($\dot{S}_{irr} = 0$), and the expansion work is recovered internally ($\dot{W}_{exp} = 0$) so the ideal or maximum *COP*, known as the Carnot value of *COP*, is given by Eq. (4) as

$$COP_{Carnot} = \frac{T_c}{T_0 - T_c}. \tag{6}$$

The efficiency (more precisely the second law efficiency) of the refrigerator is usually expressed in relation to the Carnot *COP* as

$$\eta = \frac{COP}{COP_{Carnot}}. \tag{7}$$

Figure 2 shows the results of a 1974 survey on the efficiency of various cryogenic refrigerators as well as the results of a 1998 update[1,2]. Note that the efficiency of small cryocoolers is only a few percent of Carnot, whereas for large systems the efficiency can be as high as 40 % of Carnot. This figure does not distinguish between various cycles, but such a comparison will be made later. The use of this second-law efficiency from Eq. (7) has the advantage of removing most of the temperature dependence.

The parameters and the equations introduced so far pertain to a complete refrigeration system where there are no mass flows crossing the system boundaries. Such a system is known as a closed thermodynamic system. These equations are sufficient for characterizing the complete system. For many commercial systems no further information is available. However, in order to understand the thermodynamics of the internal processes within a refrigerator or cryocooler, we must divide the system into its various components and analyze the processes within these components. As a result we now have mass flow crossing the system boundaries, and we must use the thermodynamics of open systems to analyze the processes within these systems.

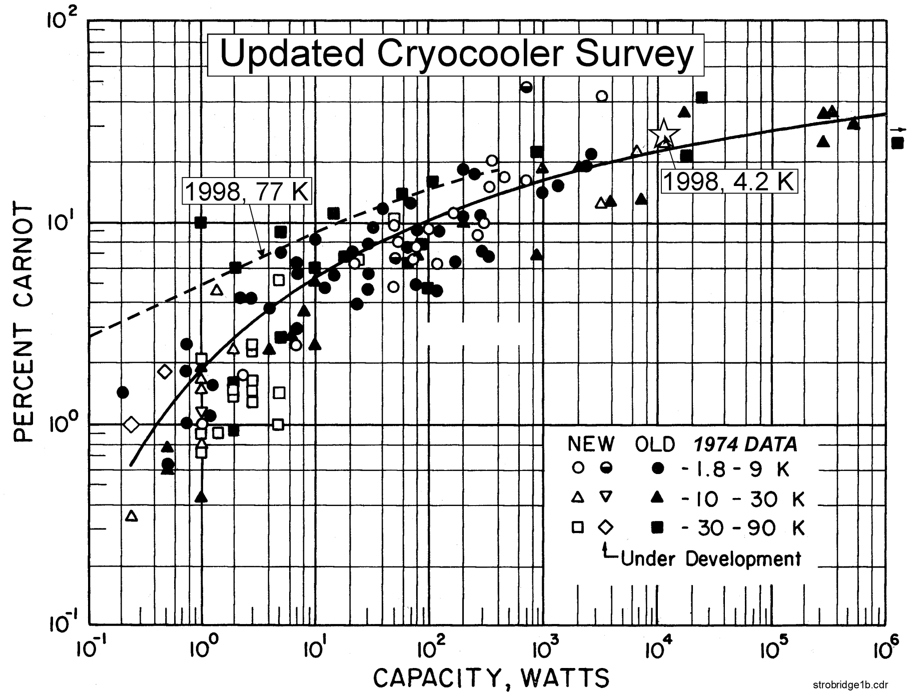

Figure 2. The second-law efficiency of cryogenic refrigerators as a function of the refrigeration capacity.

Thermodynamic fundamentals of open systems

Figure 3 shows the important thermodynamic parameters associated with an open system. The system can represent either a component of a refrigerator, as shown in Fig. 3, or a complete refrigeration system, such as liquefier with a mass flow in to and out of the system. Figure 3a shows the energy terms used in the first law and Fig. 3b shows the entropy terms used in the second law. The temperature of the reservoir transferring heat to the system is at a temperature T_j. The open system differs from the closed system by the addition of the mass flow terms $\dot{m}$ and the associated specific enthalpy h and specific entropy s of the flowing fluid at a particular

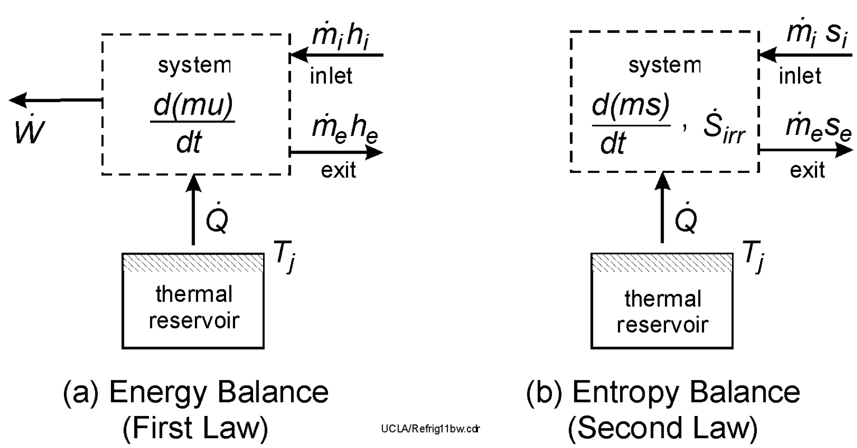

Figure 3. Schematic of a refrigerator or refrigerator component as an open thermodynamic system.

location. For the **open system** shown in Fig 3a the first law now becomes

$$\textbf{First Law}: \quad \dot{Q} = \dot{W} + (\dot{m}_e h_e - \dot{m}_i h_i) + \frac{d(mu)}{dt}. \tag{8}$$

The second law for an open system, as shown in Fig. 3b, is

$$\textbf{Second Law}: \quad \frac{\dot{Q}}{T_j} = (\dot{m}_e s_e - \dot{m}_i s_i) - \dot{S}_{irr} + \frac{d(ms)}{dt}. \tag{9}$$

The last term in each equation is zero for steady-state operation. Combining these two equations for steady-state operation yields

$$\dot{W} = \dot{m}\left[(h - T_j s)_i - (h - T_j s)_e\right] - T_j \dot{S}_{irr}. \tag{10}$$

Equation (10) shows that if the process is reversible ($\dot{S}_{irr} = 0$), the work or power produced by the process is a maximum and given by

$$\dot{W}_{rev} = \dot{m}\left[(h - T_j s)_i - (h - T_j s)_e\right]. \tag{11}$$

Equation (11) forms the basis of much of the analysis of regenerative systems, as we shall see in the following section. When irreversibilities are present the work recovered is reduced to

$$\dot{W} = \dot{W}_{rev} - T_j \dot{S}_{irr}. \tag{12}$$

The quantity $T_j \dot{S}_{irr}$ is known as the lost work $\dot{W}_{lost,j}$ referred to the temperature T_j. For heat flow $\dot{Q}_j$ from several reservoirs at some temperature T_j and several flow streams the general expression for the combined first and second laws for steady-state operation is given by[3]

$$\dot{W} + T_0 \dot{S}_{irr} = \dot{W}_{rev} = \sum_{\text{in}} \dot{m}(h - T_0 s) - \sum_{\text{exit}} \dot{m}(h - T_0 s) + \sum_j \dot{Q}_j \left(1 - \frac{T_0}{T_j}\right), \tag{13}$$

where T_0 is the temperature of the surroundings or the atmosphere and represents the reservoir for $j = 0$. The lost work $T_0 \dot{S}_{irr}$ referred to the ambient temperature represents the additional input power required to drive a refrigerator because of irreversibilities in the system.

The combination of terms (h-$T_0 s$) in Eq. (13) is called the availability function, which is different from the Gibbs function (h-Ts), where T is the temperature of the process and can be some temperature other than the surrounding temperature. The availability ψ, also known as the exergy e_x, of a process to provide the maximum reversible work is given by

$$\psi = e_x = h - h_0 - T_0(s - s_0) = \frac{\dot{W}_{max}}{\dot{m}} = \dot{w}_{max}, \tag{14}$$

where h_0 and s_0 are properties usually evaluated at the temperature and pressure of the surrounding. I prefer the use of the term availability rather than exergy because it is more descriptive. That is, for a given set of inlet conditions the fluid is 'available' to provide as much work per unit mass flow as the availability of the inlet state, providing the work is extracted reversibly in a steady-state process and exits at atmospheric pressure and temperature. The extraction of the work can be carried out with an expander piston or turbine, but in an actual case there will be some irreversibilities associated with the process. The actual work recovered would then be given by Eq. (13) if $\dot{S}_{irr}$ can be evaluated. For any steady-state process in thermal contact with the surrounding at temperature T_0 the reversible work per unit mass flow available for any inlet and outlet conditions is given by

$$\dot{w}_{rev} = \frac{\dot{W}_{rev}}{\dot{m}} = \psi_i - \psi_e = h_i - h_e - T_0(s_i - s_e). \tag{15}$$

Thermodynamics of oscillating (regenerative) systems

Types of Regenerative Systems. The thermodynamic laws discussed in the previous section deal with flow in one direction. Equations (8) and (9) can be used for instantaneous flow when the last term in each equation is included to account for the changing internal energy and entropy of the system. For regenerative cryocoolers the flow and pressure are oscillating, typically at frequencies ranging from about 1 Hz to 60 Hz. This section discusses how to apply the thermodynamics discussed in the previous section to regenerative systems. Figure 4 shows schematics of the three primary regenerative cryocoolers in current use. The working fluid is almost always helium gas. The oscillating pressure can be generated with a valveless compressor (pressure oscillator) as shown in Fig. 4 for the Stirling and pulse tube cryocoolers, or with valves that switch the cold head between a low- and high-pressure source, as shown for the Gifford-McMahon cryocooler. In the latter case a conventional compressor with inlet and outlet valves is used to generate the high- and low-pressure sources. With the Gifford-McMahon cryocooler an oil-lubricated compressor is usually used and oil removal equipment can be placed in the high-pressure line, where there is no pressure oscillation. The use of valves greatly reduces the efficiency of the system. Pulse tube cryocoolers can use either source of pressure oscillations, even though Fig. 4 indicates the use of a valveless compressor. The valved compressors are modified air-conditioning compressors, and they are used primarily for commercial applications where low cost is very important. The amplitude of the oscillating pressure may typically be anywhere from about 10 % to as high as 50 % of the average pressure. Average pressures are usually in the range of 1.5 to 3.5 MPa. The heat rejected at the warm end

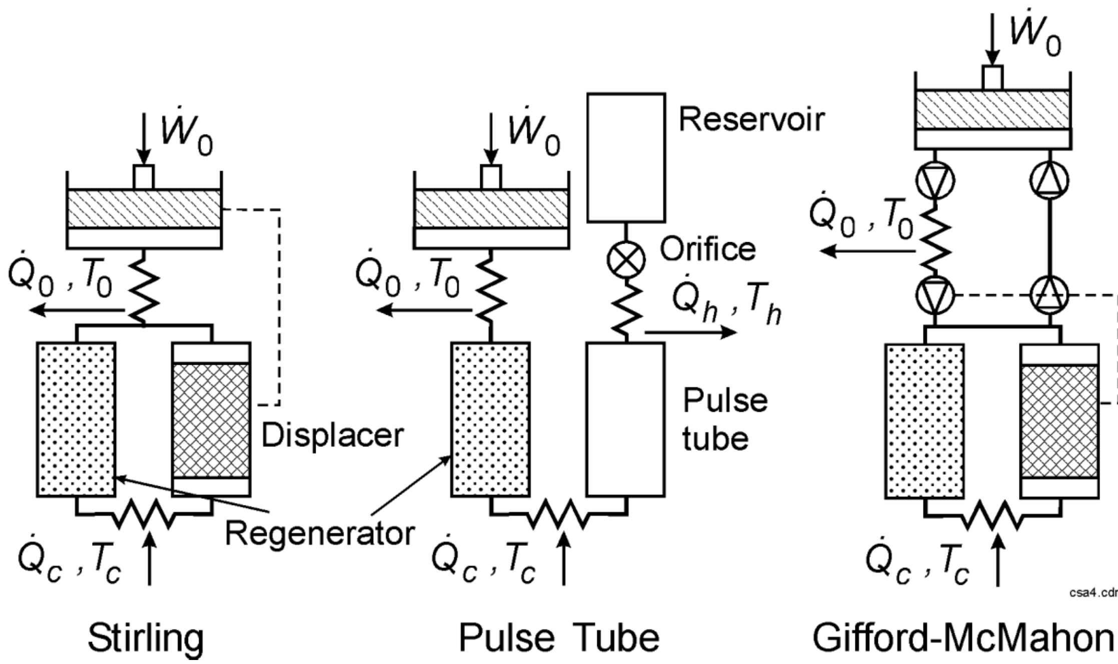

Figure 4. Schematics of the three common regenerative cryocoolers.

of the pulse tube is at the temperature T_h, which may be different from T_0.

The main heat exchanger in regenerative cycles is called a regenerator. In a regenerator, incoming hot gas transfers heat to the matrix of the regenerator, where the heat is stored for a half cycle in the heat capacity of the matrix. In the second half of the cycle the returning cold gas, flowing in the opposite direction through the same channel, absorbs heat from the matrix and returns the matrix to its original temperature before the cycle is repeated. Very high surface areas for enhanced heat transfer are easily achieved in regenerators through the use of stacked fine-mesh screen or packed spheres.

Time-Averaged Behavior. Regenerative cryocooler systems can be analyzed as a closed system by applying Eqs. (1) through (7) of the first section. Generally the small temperature oscillations on the external surfaces of regenerative cryocoolers are small enough to ignore, and we can let the last term in Eqs. (2) and (3) be zero under steady-state conditions. Similarly, the small temperature oscillations on the exterior surfaces lead to small oscillations in the heat flow superimposed on a much larger steady heat flow. We also ignore such small oscillations. The work or power undergoes large oscillations from positive to negative values whenever pistons are used. Such oscillations are characteristic of piston compressors and expanders, but normally we are interested only in the time-averaged values. Thus, even in recuperative cryocoolers, such as the Joule-Thomson, Brayton, and Claude cycles, they are analyzed by making use of time-averaged quantities, even though the oscillation amplitudes of all parameters but the power are very small.

The analysis of components of regenerative systems requires the use of thermodynamics for open systems, but now the mass flow must be treated as an oscillating parameter. Most of the information we desire about such systems is time-averaged information, so the flow parameters are then time-averaged over one cycle. The time-averaged mass flow is given by

$$\langle \dot{m} \rangle = \frac{1}{\tau}\int_0^\tau \dot{m}\,dt. \tag{16}$$

In most cases $\langle \dot{m} \rangle = 0$, although occasionally a steady (DC) component of flow may be superimposed on the oscillating (AC) component. The thermodynamics of open systems makes use of the products of mass flow and another time-varying quantity such as specific enthalpy h or specific entropy s. Each of these products may vary throughout the cycle, but have finite time-averaged values. For example, the time-average enthalpy flow is

$$\langle \dot{H} \rangle = \frac{1}{\tau}\int_0^\tau \dot{m}h\,dt = \frac{1}{\tau}\int_0^\tau \dot{m}c_p T\,dt, \tag{17}$$

and the time-averaged entropy flow is

$$\langle \dot{S} \rangle = \frac{1}{\tau}\int_0^\tau \dot{m}s\,dt. \tag{18}$$

When time averages are introduced into Eqs. (8) and (9) we obtain the first and second laws for **open oscillating systems** transferring heat with a heat sink at T_j:

$$\textbf{First Law}: \quad \langle \dot{Q} \rangle = \langle \dot{W} \rangle + \langle \dot{H} \rangle_e - \langle \dot{H} \rangle_i + \frac{d\langle U \rangle}{dt}, \tag{19}$$

$$\textbf{Second Law}: \quad \frac{\langle \dot{Q} \rangle}{T_j} = \langle \dot{S} \rangle_e - \langle \dot{S} \rangle_i - \langle \dot{S}_{irr} \rangle + \frac{d\langle S \rangle}{dt}, \tag{20}$$

where $\langle U \rangle$ and $\langle S \rangle$ are the internal energy and entropy of the system averaged over one cycle. For steady state systems these last terms in Eqs. (19) and (20) are zero. As an example, the refrigeration power $\langle \dot{Q} \rangle_c$ in a pulse tube refrigerator is given by Eq. (19) as the difference in the time-average enthalpy flows between the pulse tube and the regenerator because there is no work being extracted at the cold end. Combining Eqs. (19) and (20) for steady-state operation gives

$$\langle \dot{W} \rangle_{rev} = \langle \dot{W} \rangle + T_j \langle \dot{S} \rangle_{irr} = \left[\langle \dot{H} \rangle - T_j \langle \dot{S} \rangle \right]_i - \left[\langle \dot{H} \rangle - T_j \langle \dot{S} \rangle \right]_e. \tag{21}$$

In the previous section we discussed the concept of availability, where the reference state was the atmospheric pressure and temperature. Availability can also be used here with oscillating systems, but for a constant pressure and temperature $\langle \dot{H} \rangle_0 = 0$ and $\langle \dot{S} \rangle_0 = 0$ because there is no time-averaged mass flow. With such a reference state the maximum reversible power that can be extracted becomes

$$\langle \Psi \rangle = \langle E_x \rangle = \langle \dot{W} \rangle_{max} = \langle \dot{H} \rangle - T_0 \langle \dot{S} \rangle, \tag{22}$$

which represents the total availability or exergy (not per unit mass) at any location in the system. If a piston were placed in the fluid at any location, Eq. (22) gives the maximum power that could be recovered as long as it was done reversibly, or isothermally at the temperature T_0, and expanded to atmospheric pressure. However, this definition of availability is not so satisfying because the system in normal operation is never in equilibrium with atmospheric pressure. There is no mechanism to recover the work if the gas were to expand to atmospheric pressure. Instead a more appropriate reference state is the average or mean pressure P_m in the system. In recovering work from the oscillating system the backside of the expansion piston should be at the average pressure, unless the gas is to be expanded all the way to zero pressure. The maximum reversible power flow $\langle \dot{W} \rangle_{max}$ becomes zero when the dynamic pressure or the amplitude of the pressure oscillation

$$P_d = (P - P_m) \tag{23}$$

decreases to zero. The reference temperature is still kept at the atmospheric temperature T_0. However, we find that Eq. (22) sometimes can be even more useful when the reference temperature is the average temperature T_m at any location. The PV power that can be recovered reversibly at that temperature at that location is usually called the acoustic power, the PV power flow, or the hydrodynamic power. This acoustic power is given by

$$\langle \dot{W} \rangle_{rev} = \langle P_d \dot{V} \rangle = \frac{1}{\tau} \int_0^{\tau} (P - P_m) \dot{V} dt, \tag{24}$$

where $\dot{V}$ is evaluated at T_m. The constant reference pressure in Eq. (24) cannot be ignored because

$$\int_0^{\tau} \dot{V} dt \neq 0. \tag{25}$$

That is, the volume flow $\dot{V}$ past a fixed boundary is not conserved even though mass flow is conserved. The volume flow depends on the density, which depends on the pressure, and that varies throughout the cycle. The power flow calculated by Eq. (24) is equivalent to the power calculated from the PV diagram that a fictitious isothermal piston follows at that location. The volume flow of the piston is different from that of the volume flow past a fixed boundary and is conserved over a cycle. Because we have considered the expansion piston to be isothermal and reversible there is no additional work term associated with the volume of gas between the boundary and the piston. The volume swept by a fictitious piston is a very useful parameter in the design of pulse tubes because that swept volume must be kept significantly smaller than the volume of the pulse tube in order to provide for thermal buffering. The relationship between the acoustic power or PV power flow and the time-averaged enthalpy and entropy flows is then given by the combination of Eqs. (21) and (24) as

$$\langle P_d \dot{V} \rangle = \langle \dot{H} \rangle - T_m \langle \dot{S} \rangle. \tag{26}$$

The acoustic power does not represent an actual work term, but it indicates the potential to do reversible work if an expander piston were placed in the fluid at that location with the backside at the average pressure. Equation (26) is a rigorous expression that applies to all regenerative refrigerators and is valid for systems with losses, for real gases, and for any flow waveform.

Instantaneous Behavior. The detailed behavior within components of a regenerative refrigerator can be analyzed only by investigating the behavior within one cycle. For such analyses we must use the general form of the first law given by Eq. (8). A full analysis also requires the use of differential equations for the conservation of mass as well as momentum. These four differential equations are solved simultaneously using finite-difference techniques. In most cases a one-dimensional solution is adequate, although higher-order flow effects, particularly in the pulse tube, may require a two-dimensional approach. The solution to these equations shows how the losses depend on detailed geometry and properties of the regenerator and on the operating conditions in the system. For locations where no work crosses the boundary the four equations are:

Conservation of Energy (Gas)

$$\frac{h_t}{r_h}(T_m - T) = \frac{\partial}{\partial x}\left[\left(\dot{m}/A_g\right)h\right] - \frac{\partial}{\partial x}\left[k_g \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial t}(\rho u); \tag{27}$$

Conservation of energy (Matrix)

$$\frac{h_t}{r_h}(T - T_m) = -\left[\frac{1-n_g}{n_g}\right]\frac{\partial}{\partial x}\left[k_m \frac{\partial T_m}{\partial x}\right] + \left[\frac{1-n_g}{n_g}\right]\rho_m c_m \frac{\partial T_m}{\partial t}; \tag{28}$$

Conservation of Mass

$$\frac{\partial}{\partial x}\left[\frac{\dot{m}}{A_g}\right] = -\frac{\partial \rho}{\partial t}; \tag{29}$$

Conservation of Momentum

$$-\frac{\partial P}{\partial x} = \left[\frac{\dot{m}}{A_g}\right]\left[\frac{|\dot{m}|}{A_g}\right]\frac{f_r}{2 r_h \rho} + \frac{\partial}{\partial x}\frac{1}{\rho}\left[\frac{\dot{m}}{A_g}\right]^2 + \frac{\partial}{\partial t}\left[\frac{\dot{m}}{A_g}\right]. \tag{30}$$

In these equations r_h is the hydraulic radius, A_g is the gas cross-sectional area, x is the axial position, k is the thermal conductivity of the gas, k_m is the axial thermal conductivity of the matrix, n_g is the porosity of the matrix, $\rho_m c_m$ is the volumetric heat capacity of the matrix, and f_r is the Fanning friction factor. The last term in Eq. (30) causes a component of pressure gradient in phase with the acceleration of the working fluid mass and is referred to as the inertance effect in analogy with the inductive effect in electrical circuits. It is made use of in the inertance tube of pulse tube refrigerators.

The equation for conservation of mass, Eq. (29), is often integrated over the length of a particular component in order to relate the mass flow rate at the two ends of the component. For any isothermal component with a perfect gas this integrated equation relates the hot end mass flow to the cold end mass flow by

$$\dot{\boldsymbol{m}}_{\boldsymbol{h}} = \dot{\boldsymbol{m}}_{\boldsymbol{c}} + \frac{\dot{\boldsymbol{P}}V}{RT_a}, \tag{31}$$

where the parameters in bold type are time varying parameters with arbitrary phase relationships, V is the gas volume of the element, R is the gas constant per unit mass, and T_a is the average temperature in the component. For a regenerator that spans a large temperature difference, we typically use the log-mean average for T_a. For the end components, such as the expander and the compressor, there is no mass flow crossing the moving system boundary, but there is work crossing the boundary at that location. Mass conservation applied to the expander and with the assumption of isothermal expansion yields

$$\dot{\boldsymbol{m}}_{\boldsymbol{c}} = \frac{\boldsymbol{P}\dot{\boldsymbol{V}}_e}{RT_c} + \frac{\dot{\boldsymbol{P}}\boldsymbol{V}_{\boldsymbol{e}}}{RT_c}, \tag{32}$$

where $\boldsymbol{V}_{\boldsymbol{e}}$ is the instantaneous expander gas volume. For relatively small pressure amplitudes such as those typical in Stirling refrigerators and Stirling-type pulse tube refrigerators, Eq. (32) can be approximated by

$$\dot{\boldsymbol{m}}_{\boldsymbol{c}} = \frac{P_m \dot{\boldsymbol{V}}_{\boldsymbol{e}}}{RT_c} + \frac{\dot{\boldsymbol{P}}V_E}{2RT_c}, \tag{33}$$

where V_E is the total swept volume of the expander, and the average volume during the cycle is taken as half that volume. For the isothermal compressor and with the proper sign convention for the flow direction we have the equivalent relationship

$$\frac{-P_m \dot{\boldsymbol{V}}_{\boldsymbol{co}}}{RT_{co}} = \dot{\boldsymbol{m}}_{\boldsymbol{co}} + \frac{\dot{\boldsymbol{P}}V_{CO}}{2RT_{co}}, \tag{34}$$

where V_{CO} is the total swept volume of the compressor and $\boldsymbol{V}_{\boldsymbol{co}}$ is the instantaneous compressor gas volume.

For an adiabatic component such as the pulse tube, we can derive a similar expression, but we start first with the first law for an open system, Eq. (8). Because no work is being done in the pulse tube and because there is no heat transfer in the adiabatic process, we are left with the following simplified equation when gas conduction is ignored

$$\frac{d(\rho V_t u)}{dt} + \dot{m}_{ht} h_{ht} - \dot{m}_c h_c = 0, \tag{35}$$

where the subscript *ht* refers to the hot end of the pulse tube, and V_t is the volume of the pulse tube. We now assume ideal gas behavior in order to use the following relationships

$$h = c_p T; \qquad u = c_v T = c_v \frac{P}{\rho R}, \tag{36}$$

where c_p and c_v are the specific heats of the gas at constant pressure and at constant volume, respectively. When the relationships of Eq. (36) are substituted into Eq. (35) we have

$$\dot{\boldsymbol{m}}_{\boldsymbol{c}} = (T_{ht} / T_c)\dot{\boldsymbol{m}}_{\boldsymbol{ht}} + \frac{\dot{\boldsymbol{P}}V_t}{\gamma R T_c}, \tag{37}$$

where γ is the ratio of the specific heats. Note that the temperature used in the last term is the temperature of the cold end and not the average temperature.

Harmonic Approximations. In the Stirling refrigerator and the Stirling-like pulse tube refrigerator the pressure, flow, and temperature oscillations are quite close to sinusoidal behavior. In these refrigerators the use of harmonic (sinusoidal) approximations in the equations discussed here can quite accurately describe their behavior and eliminate the need for numerical integration in the time domain. When the pressure and flow vary sinusoidally the time-averaged quantities such as the PV power flow are given by

$$\langle P_d \dot{V} \rangle = (1/2)|P_d||\dot{V}|\cos\theta = (1/2)P_1\dot{V}_1 \cos\theta = (1/2)RT_m \frac{P_1}{P_m}\dot{m}_1 \cos\theta, \tag{38}$$

where the subscript 1 refers to amplitude of the sinusoidal quantity, the subscript *m* refers to the mean value, and θ is the phase angle between the pressure and flow. Equation (38) shows that for a given amplitude of mass flow the maximum PV power flow through a regenerator can be achieved when the pressure and the flow are in phase. Of course, that phase relationship can occur at only one location within the regenerator (generally the average or midpoint) because the phase of the mass flow varies considerably with position. Minimizing the amplitude of mass flow in the regenerator is important because the losses (both pressure drop and thermal) are proportional to the amplitude of mass flow. Losses within an expander or a pulse tube are only weak functions of mass flow amplitudes, so the phase optimization should occur within the regenerator rather than at the cold end.

The use of the harmonic approximation allows for analytical solutions to most of the equations given here to be obtained by utilizing complex variables to describe the harmonic behavior. Thermoacoustic theories make use of the harmonic approximation and rely on complex variables to solve equations. The harmonic approximations also allow the use of phasors to show graphically the solution to various equations and the relative phase relationships between different oscillating quantities. Figure 5 shows example phasor diagrams representing the simultaneous solution of Eqs. (31), (33), (34), and (37) in various components of a Stirling and a pulse tube refrigerator. The compressor for both cases is assumed to be isothermal and the Stirling expander is also assumed to be isothermal. Processes in the pulse tube component are assumed to be adiabatic. The pulse tube refrigerator modeled in Fig. 5 is a simple orifice type, in which the mass flow at the orifice is in phase with the pressure in the pulse tube. The use of a secondary orifice or an inertance tube would cause the mass flow to lag the pressure at the warm end of the pulse tube and provide a more favorable phase relationship in the regenerator. All of the vectors (phasors) in Fig. 5 rotate about the axis origin at the operating frequency ω and maintain the same phase relationship with each other at all times. The projection of each phasor

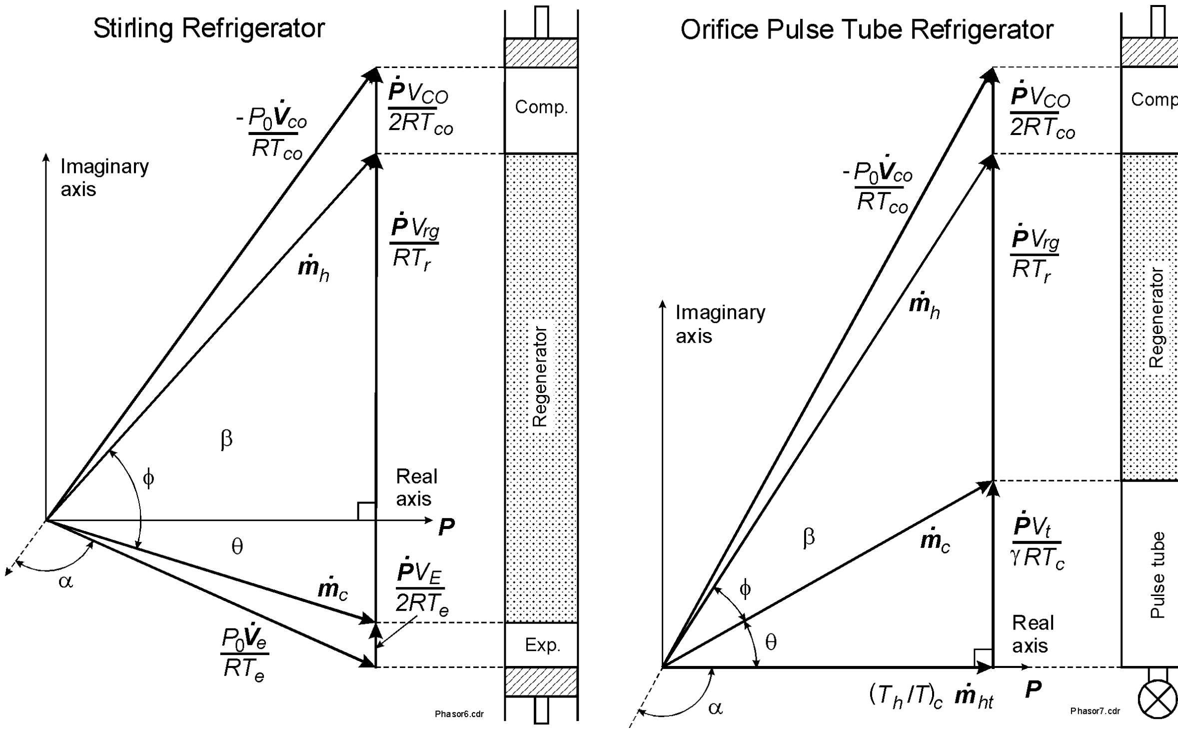

Figure 5. Phasor diagrams for the Stirling and pulse tube refrigerators, showing graphically the solution to the equation for conservation of mass. The equation for conservation of energy is also used for the pulse tube.

onto the real axis gives the sinusoidally varying values for each parameter. With the harmonic approximation, the time derivative of a parameter, for example $\dot{\boldsymbol{P}}$, leads the parameter, for example $\boldsymbol{P}$, by π/2 or 90°. The graphical solutions shown in Fig. 5 relate the system pressure amplitude to the geometry of the system and the temperatures at various locations. The phasor diagrams of Fig. 5 are also a useful indicator of the PV power flow at various locations. According to Eq. (38) the PV power flow is proportional to $P_1\dot{m}_1\cos\theta$, which is simply the dot product of these two vectors (phasors). For a given pressure amplitude the PV power flow is proportional to the projection of the mass phasor on the pressure phasor. As Fig. 5 shows the flow rate at the face of the compressor piston is different from the flow rate past the fixed boundary at the output of the compressor. However, because the projection of both flows onto the pressure phasor is the same, the PV power flow at both locations is the same as long as the temperature is the same.

Energy flows in regenerative refrigerators

We now investigate the behavior of enthalpy and entropy flow in two ideal elements, a perfectly isothermal element and a perfectly adiabatic element. An ideal pulse tube refrigerator consists of a collection of such elements. Ideal heat exchangers and regenerators are represented by isothermal elements, in which the temperature at any location does not change with time. The pulse tube and some compressor cylinders are represented by adiabatic elements, in which there is no heat transfer between the gas and the element walls even on an instantaneous basis. For enthalpy flow in an isothermal element where T is a constant at any location, we see from Eq. (17) that for an ideal gas, where c_p and h are independent of pressure, $\langle\dot{H}\rangle = 0$ whenever

$\langle \dot{m} \rangle = 0$. For a real gas h is a function of pressure and $\langle \dot{H} \rangle$ is finite even for a perfect heat exchanger or regenerator. Because s is a function of pressure at a constant temperature, even for an ideal gas, $\langle \dot{S} \rangle$ is finite in an isothermal element. However, in an ideal regenerator there is no time-averaged heat flow to the system, so from Eq. (20) $\langle \dot{S} \rangle$ is a constant throughout the regenerator. In an ideal adiabatic element the entropy s in Eq. (18) is independent of time because there is no instantaneous heat transfer or irreversible process to cause it to change. Thus, $\langle \dot{S} \rangle$ = 0 in the ideal adiabatic element. Because $\langle \dot{Q} \rangle$ = 0 in an ideal adiabatic element, $\langle \dot{H} \rangle$ will be constant throughout the element according to Eq. (19) as long as there is no work being done along the element. An example is the pulse tube element where no work is being done. We summarize the behavior of these two ideal elements here as

$$\textbf{ideal isothermal element}: \quad \langle \dot{H} \rangle = 0, \quad \langle \dot{S} \rangle = \text{constant}; \tag{39}$$

$$\textbf{ideal adiabatic element}: \quad \langle \dot{H} \rangle = \text{constant}; \quad \langle \dot{S} \rangle = 0. \tag{40}$$

Figure 6 shows a schematic of a pulse tube refrigerator and the behavior of the energy flows at various locations. The upper set of curves is for an ideal refrigerator whereas the lower set of curves is for a real refrigerator with losses. Beginning at the left we see that the piston introduces PV power into the system. The system boundary is at the face of the piston, and

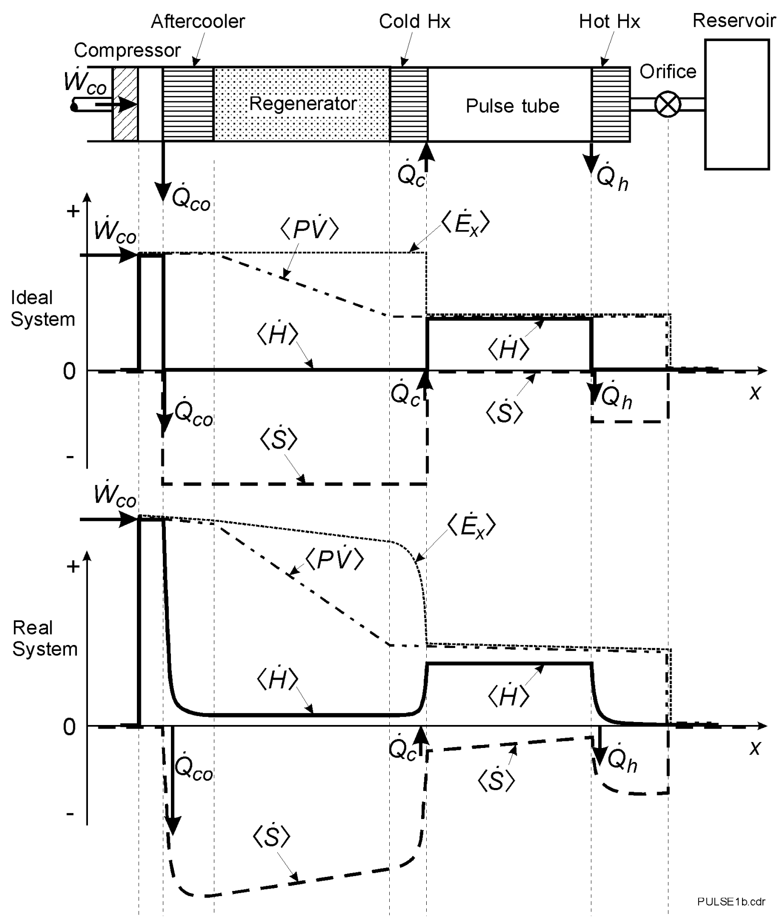

Figure 6. The work, heat, PV power, enthalpy, and entropy flows in an ideal and a real pulse tube refrigerator. Positive flow is to the right.

because the system boundary moves with a volume velocity of $\dot{V}$ there is a true thermodynamic work term at this system boundary. The moving piston causes an oscillating mass flow and pressure within the helium working fluid that gives rise to enthalpy, entropy, and exergy (or availability) flows. Our sign convention for flow terms is positive for flow to the right. To the right of the piston inside the compressor there exists within the helium working fluid an acoustic power flow, commonly called a PV power flow, that is equal to the input PV power. It moves to the right (positive) and is related to the enthalpy and entropy flows by Eq. (26). In the ideal pulse tube system (no pressure drops) $\langle P_d\dot{V}\rangle$ does not change until it begins to flow through the regenerator. The regenerator is an isothermal element at any location, but the temperature changes from one end to the other. For an ideal gas the specific volume is proportional to temperature, which suggests that $\langle P_d\dot{V}\rangle$ varies in proportion to temperature through the regenerator. Also, in the perfect regenerator with an ideal gas, $\langle\dot{H}\rangle = 0$ and $\langle\dot{S}\rangle$ is a constant from one end to the other. The negative value of $\langle\dot{S}\rangle$ indicates that it travels to the left toward the source of the work input. Equation (26) then shows that because $\langle P_d\dot{V}\rangle = -T_m\langle\dot{S}\rangle$ in this case, $\langle P_d\dot{V}\rangle$ must vary in proportion to the average temperature T_m at any location. In the perfectly adiabatic pulse tube component $\langle\dot{S}\rangle = 0$ and $\langle\dot{H}\rangle$ is a constant from one end to the other (no heat or work input). Equation (26) then shows that $\langle P_d\dot{V}\rangle = \langle\dot{H}\rangle$ in this case and must be constant through the pulse tube component. The unique function of the pulse tube component is its ability to transfer the acoustic power across a temperature gradient with no change in value. A more descriptive name for the pulse tube component might be a work transfer tube or a thermal buffer volume, but custom has given it the name 'pulse tube' even though smooth sinusoidal oscillations of pressure and flow can occur within it. The enthalpy flow and the entropy flow both change at the heat exchangers in accordance with the first and second laws given by Eqs. (19) and (20) because of the heat flow at those locations. However, the acoustic power does not change at those locations since the change in the enthalpy and entropy terms in Eq. (26) cancel each other. The heat absorbed at the cold heat exchanger and the heat rejected at the warm heat exchanger are simply equal to $\langle\dot{H}\rangle$ in the pulse tube. These heat flows at the two ends are also related to the entropy changes there from the second law, Eq. (20). It is also the second law that shows that $\langle\dot{S}\rangle$ is a constant throughout a perfect regenerator, which then relates the heat absorbed at the cold end to the heat rejected at the aftercooler by the expression

$$\frac{\langle\dot{Q}_0\rangle}{T_0} = \frac{\langle\dot{Q}_c\rangle}{T_c}. \tag{41}$$

The time-averaged availability or exergy flow $\langle E_x\rangle$ given by Eq. (22) is also shown in this figure, with the reference pressure being the average or mean pressure. In the ideal case it changes only with heat flow at some temperature other than ambient.

The lower set of curves in Fig. 6 show the energy flows for a real system where there are losses that generate entropy throughout the system. This example is for the same net refrigeration power $\dot{Q}_c$ as for the ideal case. We see that the losses everywhere in the system lead to a larger input power at the compressor and a larger heat rejection at the aftercooler. The heat flows at the heat exchangers are also shown to occur over the entire length of the heat exchanger instead of at the boundary for the ideal case. The finite enthalpy flow in the regenerator also leads to an increased heat flow from the heat exchanger at the warm end of the pulse tube. Losses in the pulse tube and the regenerator show up as changes in the entropy and exergy flows through those components. In the case of the Stirling refrigerator an expander piston or displacer is placed in the system just to the right of the cold heat exchanger and part way into the 'pulse tube' in Fig. 6. In that case there is no flow past that system boundary and all the flow terms such as acoustic power, enthalpy flow, and entropy flow are zero beyond that boundary with the expander. Real power $\langle \dot{W}_{exp} \rangle$ is extracted from the system by the expander (moving boundary) and is equal to the acoustic power $\langle P_d \dot{V} \rangle$ at that location only if the expansion is reversible and isothermal. According to the first law the actual power extracted is equal to the enthalpy flow entering the expansion space if it is an adiabatic process. The heat absorbed still occurs at the heat exchanger in this adiabatic process. For an isothermal expansion the expander face must be located within the cold heat exchanger, in which case the enthalpy flow there is zero in the ideal case and the first law shows that the refrigeration power is equal to the power extracted by the expander, which in turn is equal to the acoustic power for a reversible process. In practice the irreversibilities associated with the expansion process lead to extracted power that may be only about 85 % of the acoustic power $\langle P_d \dot{V} \rangle$.

Unlike recuperative systems, which have a steady mass flow in one direction, there is no time-averaged mass flow in regenerative systems. Thus, a question often arises as to where the heat goes that enters the cold end. Some say it is transported by the enthalpy to the heat exchanger at the warm end of the pulse tube. Others will say the regenerator 'pumps' the heat from the cold end to the warm end of the regenerator and rejects it at the aftercooler. In that case it must travel with the entropy, which is flowing from the cold end toward the compressor. Those who argue this second point often multiply the entropy flow by the local average temperature along the length of the regenerator, $T_m \langle \dot{S} \rangle$, and call it the heat flow from the cold to the warm end of the regenerator that increases in proportion to the temperature. If one observes thermodynamic principles closely, neither argument is entirely correct. The first point I wish to make here is that heat can flow only from a higher temperature to a lower temperature. That is the only type of heat recognized by the first and second law of thermodynamics. Heat can flow from a heater to the cold gas inside the cold heat exchanger because of a small temperature gradient between the heater and the gas. That heat then changes the temperature of the gas flowing through the heat exchanger and increases both its enthalpy and entropy. At that point 'heat' has vanished and has been converted into the altered gas properties. Both enthalpy flow toward the pulse tube warm end and entropy flow toward the regenerator warm end occur simultaneously as a result of the heat input to the oscillating gas at the cold end. There is no time-averaged 'heat' flow associated with either the enthalpy or entropy flows. It is only at the warm ends where thermal contact to the surrounding is provided that there is heat flow from the warmer gas there to the cooler surroundings. The heat input at the cold end alters the gas properties so as to cause heat to be rejected at both the warm end of the pulse tube and the warm end of the regenerator. The only time-averaged heat flows in the system occur between the gas and the heat exchangers. When system boundaries are drawn around either the regenerator or the pulse tube, there are no $\langle \dot{W} \rangle$ or $\langle \dot{Q} \rangle$ terms if axial conduction is ignored. The energy flows associated with $\langle \dot{H} \rangle$ and $\langle \dot{S} \rangle$ have the potential to transfer heat with the surrounding whenever there is a change from an isothermal to an adiabatic element to cause these flows to change. The product $T_m\langle\dot{S}\rangle$ in a regenerator could be considered the potential to transfer heat with the surrounding wherever the proper elements (heat exchanger and an adiabatic element to block entropy transport) are inserted in the system to transfer the heat. Likewise, the flow has the potential, or availability, to do real work whenever a moving piston is inserted into the system at some location.

To the right of the warm heat exchanger shown in Fig. 6 is a flow impedance. It can be an orifice, a capillary, a sintered plug, or an inertance tube that makes use of the inertia of the oscillating helium to shift the phase between the mass flow and the pressure.[4-7] This impedance is an intrinsic loss element in which the pressure drop causes the loss of the acoustic power through an irreversible production of entropy. In the Stirling refrigerator this available work is recovered at the cold end by use of a displacer, which feeds the recovered work back into the system at the other end of the displacer. In a pulse tube refrigerator the entropy flow changes in the flow impedance because of the irreversible production of entropy during the pressure drop. In the reservoir the dynamic pressure P_d is zero, so $\langle P_d\dot{V}\rangle$ is zero there. The reservoir is generally an adiabatic element so $\langle\dot{S}\rangle$ will also be zero at the entrance to the reservoir.

With regard to the flow impedance there is sometimes a misconception in regards to the energy flow terms. Contrary to some beliefs, the pressure drop within the flow impedance does <u>not</u> cause the impedance to heat in analogy to the flow of current through a resistor. Instead, the pressure drop generates irreversible entropy $\dot{S}_{irr}$, which is different from heat, and according to Eq. (20) also changes the entropy flow at the orifice as shown in Fig. 6. For steady flow in one direction a large pressure drop in an ideal gas results in no temperature change. For a real gas the pressure drop usually results in a temperature reduction (Joule-Thomson effect). Using the first law for an open system, Eq. (8), we see that there can be no temperature change in steady flow unless the specific enthalpy is a function of pressure. For an ideal gas the enthalpy is independent of pressure. Whether this impedance heats under oscillating flow is very much dependent on its geometry. The first law for an open oscillating system, Eq. (19) shows that in order to have any heat transfer from the impedance/reservoir system, there must be some time-average enthalpy transport $\langle\dot{H}\rangle$ at the entrance to the impedance. If the impedance is an isothermal element such as a capillary or hole of small diameter (orifice), $\langle\dot{H}\rangle$ is nearly zero as discussed earlier and no heat is rejected in the impedance. Instead the heat is rejected in the warm heat exchanger. If the impedance is approximately an adiabatic element, such as a long inertance tube of large diameter, $\langle\dot{H}\rangle$ can be large and approach the value of the acoustic power flow. The inertance tube will then begin to heat until the temperature gradient in it is large enough to reduce $\langle\dot{H}\rangle$ to zero. Cooling the inertance tube/reservoir will remove an amount of heat equal to $\langle\dot{H}\rangle$. Analysis of the warm heat exchanger by use of the first law, Eq. (19), shows that with enthalpy transport in the inertance tube, the amount of heat that is rejected at the warm heat exchanger becomes

$$\langle\dot{Q}_h\rangle = \langle\dot{H}\rangle_{pt} - \langle\dot{H}\rangle_{imp}, \tag{42}$$

where $\langle\dot{H}\rangle_{pt}$ is the enthalpy flow in the pulse tube and $\langle\dot{H}\rangle_{imp}$ is the enthalpy flow in the impedance or inertance tube. For sufficiently large $\langle\dot{H}\rangle_{imp}$ the heat rejected at the warm heat exchanger approaches zero and its only function then is to provide flow straightening. Thus, its design may be changed. This same situation may occur in the use of an orifice if a relatively large gas volume is placed between the orifice and the warm heat exchanger. This heating effect in the impedance can become particularly large if the gas volume within it or in a connecting tube between the heat exchanger and the impedance becomes somewhat larger than the swept volume in a cycle at that location. In that case this volume begins to behave just like a pulse tube with its ability to absorb heat at the end toward the compressor and reject heat at the opposite end.

If the enthalpy flow in the pulse tube is converted to work at the warm end of the pulse tube by the use of a moving piston,[8,9] then no heat will be rejected according to the first law. Thus, there is no need for a warm heat exchanger in that situation, although some flow straightening may be required. When the expander piston is placed at the cold end, as in the Stirling refrigerator, work is recovered at that location to balance the heat input. There is no enthalpy flow to the right of the expander because there is no gas flow there. In the case of a displacer used for the expander the work is transmitted through the displacer to the warm end, where it then is reintroduced to the gas as a PV power flow. That PV power flow and mass flow add to that from the compressor, which reduces the required net input power by the amount of power recovered at the cold end with the displacer.

Losses in regenerative refrigerators

Much of our previous discussion has been for ideal systems where losses have been ignored. We have included the effect of irreversible production of entropy $\dot{S}_{irr}$ in the second-law analyses discussed previously, but we have not discussed how to calculate $\dot{S}_{irr}$ from various losses in regenerative cryocoolers. Understanding the losses could lead to methods to reduce them and improve efficiencies of cryocoolers. Figure 7 compares the efficiencies of various types of cryocoolers for a cold temperature of 80 K.[10] This figure shows that efficiencies increase with size and that regenerative cryocoolers tend to have higher efficiencies than recuperative cryocoolers. Pulse tube cryocoolers driven with Stirling-type (valveless) compressors have the

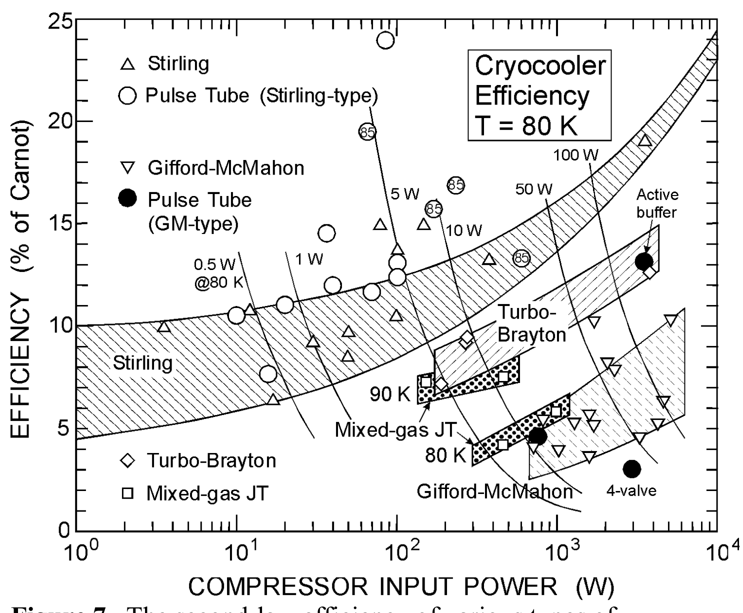

Figure 7. The second-law efficiency of various types of cryocoolers at 80 K as a function of the compressor input power. Curves for constant refrigeration power are also shown.

highest efficiencies, followed closely by Stirling cryocoolers. Gifford-McMahon (GM) cryocoolers and GM-type pulse tube cryocoolers have significantly lower efficiencies, which can be attributed to the losses in the valves and the valved compressors. Typically a valved compressor or even a GM scroll compressor is only about 50 % efficient in converting electrical to steady-flow PV power. The use of additional valves to obtain oscillating PV power introduces more losses so the overall efficiency of converting electrical to oscillating PV power may be as low as 35 to 40 %. Good Stirling-type compressors can be as high as 85 % efficient in converting electrical to oscillating PV power. Thus, much of the improved efficiency in regenerative cryocoolers is a result of the compressor. The rest of this section deals with losses in the cold head and how to calculate $\dot{S}_{irr}$ from some of these losses.

For heat flow across a finite temperature difference ΔT, such as in a heat exchanger, we have

$$\langle \dot{S}_{irr} \rangle = \frac{1}{\tau}\int_0^\tau \frac{\dot{Q}}{T}\frac{\Delta T}{T_f}dt = \frac{\langle \dot{Q}\Delta T \rangle}{TT_f}, \tag{43}$$

where T is the external temperature and T_f is the internal fluid temperature. A small pressure drop ΔP leads to an irreversible production of entropy:

$$\langle \dot{S}_{irr} \rangle = \frac{R}{P_0 \tau}\int_0^\tau \dot{m}\Delta P dt, \tag{44}$$

where R is the gas constant per unit mass and P_0 is the average pressure. This pressure drop leads to a lost power at ambient temperature (additional compressor power) of

$$\langle \dot{W}_{lost} \rangle = T_0 \langle \dot{S}_{irr} \rangle. \tag{45}$$

In the pulse tube refrigerator the pressure drop in the flow impedance is an intrinsic loss associated with the refrigerator. The PV power that could have been recovered by a piston as given in Eq. (24) is instead dissipated irreversibly. For an ideal gas and all other parts of the pulse tube refrigerator being ideal the lost work at the warm end of the pulse tube is the same as the PV power flow at the cold end. Thus, we have

$$\frac{\langle \dot{W}_{lost} \rangle}{\dot{W}_{co}} = \frac{T_c}{T_0}. \tag{46}$$

Substituting this value into Eq. (4) and letting $\dot{W}_{exp} = 0$ shows that the COP of the ideal pulse tube refrigerator is

$$COP_{pt} = \frac{T_c}{T_0}, \tag{47}$$

which agrees with the result given by Kittel.[11]

There is another intrinsic loss mechanism associated with the pulse tube refrigerator whenever finite pressure amplitudes are used. This loss occurs at the boundary between an isothermal element and an adiabatic element, such as the pulse tube. The oscillating temperature of the gas in the pulse tube causes a finite temperature difference to occur at the heat exchangers, which results in an irreversible production of entropy. Such boundary losses have been analyzed previously.[12-15] For an acoustic approximation, where the sinusoidal pressure amplitude P_1 is less than about 30 % of the average or mean pressure P_m, Swift gives an approximate expression[15] for this loss that can be rewritten as

$$\frac{\langle P_d \dot{V} \rangle_{lost}}{\langle P_d \dot{V} \rangle} = \frac{8}{3\pi} \frac{\gamma - 1}{\gamma} \frac{P_1}{P_m} \cos\theta, \qquad (48)$$

where γ is the ratio of specific heats and θ is the phase between the volume flow and the pressure at the interface. This expression is good to within 10 % even for $P_1/P_m = 0.3$. For a typical value of $P_1/P_m = 0.13$ in a pulse tube, this fractional loss is 4.4% for $\theta = 0$. The loss occurs at both ends of the pulse tube, so the total loss would be 8.8 %, except that the phase at the warm end of the pulse tube could be as much as 60° to 70°, so the total loss can be somewhat less than 8.8 %.

Other losses within the pulse tube are due to viscous and thermal dissipation brought about by the interaction between the boundary layer and the tube walls that causes some deviation from perfect adiabatic behavior. These losses become more serious in small pulse tubes. Several thermoacoustic models[16-18] exist that describe such losses in detail. For the simple case of no temperature gradient the viscous and thermal losses are[15]

$$\langle P_d \dot{V} \rangle_{loss} = T_m \dot{S}_{irr} = \frac{1}{4} \rho_m v_1^2 \delta_v A\omega + \frac{1}{4} \frac{P_1^2}{\gamma P_m} (\gamma - 1) \delta_k A\omega, \qquad (49)$$

where ρ_m is the density at the mean temperature and pressure, v_1 is the amplitude of the average velocity, δ_v is the viscous penetration depth, δ_k is the thermal penetration depth, and A is the surface area. The fractional loss of PV power in a tube of radius r due to viscous and thermal effects in the boundary layer with no axial temperature gradient is given by

$$\frac{\langle P_d \dot{V} \rangle_{loss}}{\langle P_d \dot{V} \rangle} = \frac{\gamma(\delta_v / r)(x_1 / L)(L / \lambda)^2}{(P_1 / P_m)\cos\theta} + \frac{(P_1 / P_m)(\delta_t / r)[(\gamma - 1)/\gamma]}{(x_1 / L)\cos\theta}, \qquad (50)$$

where L is the length of the tube, x_1 is the amplitude of the gas displacement during the sinusoidally oscillating flow, and λ is the wavelength of sound inside the tube.

Such losses as those due to turbulence and acoustic streaming[15, 19, 20] within the pulse tube are extremely difficult to model. Thus, we generally rely on an empirical factor in the design of pulse tube refrigerators. We call this factor the figure of merit or effectiveness of the pulse tube. It pertains to only the pulse tube component and makes use of Eq. (26). As mentioned earlier a perfect pulse tube is a purely adiabatic element with no time-averaged entropy flow. According to Eq. (26) the enthalpy flow is then equal to the PV power flow in the pulse tube, which can be measured relatively easy at the warm end of the pulse tube. Any losses lead to the generation of irreversible entropy $\langle \dot{S}_{irr} \rangle$, which is always positive, and according to Eq. (20) leads to a negative entropy flow (flow direction towards the compressor) in the gas. According to Eq. (26) this negative entropy flow reduces the time-averaged enthalpy flow in the pulse tube. The reduced enthalpy flow reduces the refrigeration power according to Eq. (19) from the first-law analysis. We then define the effectiveness of the pulse tube component as

$$\varepsilon_{pt} = \frac{\langle \dot{H} \rangle}{\langle P\dot{V} \rangle} = 1 - \frac{T_m \langle \dot{S}_{irr} \rangle}{\langle P\dot{V} \rangle}. \qquad (51)$$

Early measurements[21] have shown that this value depends on the pressure ratio and can vary from about 0.65 to about 0.85. Values of about 0.88 at 90 K were found more recently on a pulse tube oxygen liquefier of high efficiency.[22] A value of 0.96 was found in a very large system operating at 120 K, where a tapered pulse tube was used to eliminate acoustic streaming.[23]

Conclusions

Thermodynamics has been a powerful tool for over a century to relate heat and work to each other and to the properties of fluid flow within the element being studied. However, it has only been in the last 10 or 15 years that the application of thermodynamics to components of regenerative systems has been well understood. In most cases the energy terms (heat and work) communicating with the environment are of interest only in a time-averaged sense, that is over time intervals much longer than the period of oscillations within regenerative systems. Thus, the traditional thermodynamics of open systems applies to regenerative systems as long as time-averaged values are used for the heat and work energy terms as well as for the fluid flow properties such as enthalpy and entropy flow. However, the calculation of losses usually requires the application of thermodynamics of open systems on an instantaneous basis, which becomes much more complex. Losses within regenerators and pulse tubes can be calculated accurately only by use of sophisticated numerical models. The assumption of sinusoidal behavior (harmonic approximation) for the flow and pressure within regenerative systems can greatly simplify the calculations and often lead to analytical expressions for some losses. Much progress has been made in the last 10 to 15 years on detailed analyses of losses within regenerative systems. Regenerative cryocoolers, such as Stirling and pulse tube cryocoolers, are the most efficient cryocoolers, at least for small systems and for temperatures above about 20 K. Considerably more theoretical and experimental work is still required to find ways to reduce these losses and to improve the efficiency of regenerative refrigerators even further.

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