APPROXIMATE DESIGN METHOD FOR SINGLE STAGE PULSE TUBE REFRIGERATORS

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ABSTRACT

An approximate design method is presented for the design of a single stage Stirling type pulse tube refrigerator. The design method begins from a defined cooling power, operating temperature, average and dynamic pressure, and frequency. Using a combination of phasor analysis, approximate correlations derived from extensive use of REGEN3.2, a few 'rules of thumb,' and available models for inertance tubes, a process is presented to define appropriate geometries for the regenerator, pulse tube and inertance tube components. In addition, specifications for the acoustic power and phase between the pressure and flow required from the compressor are defined. The process enables an appreciation of the primary physical parameters operating within the pulse tube refrigerator, but relies on approximate values for the combined loss mechanisms. The defined geometries can provide both a useful starting point, and a sanity check, for more sophisticated design methodologies.

KEY WORDS: pulse tube refrigerators, design

INTRODUCTION

Two 50-minute class sessions of the Cryogenics course at the University of Wisconsin – Madison are devoted to the topic of pulse tube refrigerators. During this time, the objective of the lectures and discussion is to introduce students who are completely

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unfamiliar with the topic, to the design process of the pulse tube refrigerator and the physical parameters that define, constrain and optimize its geometry. Various authors provide well-developed numerical models for designing pulse tube refrigerators that simultaneously solve the equations of mass, momentum, and energy conservation. [1-3] However, while such models are recommended for those interested in an accurate design or commercial product, they are impractical as a means to introduce a physically intuitive justification for pulse tube design. The material presented in this report summarizes the lectures presented to the first year graduate students in the Cryogenics course and provides a step-by-step procedure for an approximate design of the pulse tube refrigerator. This approach relies on design charts for the regenerator and inertance tube that have been developed independently through the use of REGEN3.2, and inertance tube models [4-8]. It utilizes optimized phase relations between the sinusoidal mass flow and pressure waves in a manner similar to that presented by Hoffman & Pan[9] and Radebaugh[10], and makes use of an empirical 'rule-of-thumb' to determine the pulse tube volume. Most significantly, the approach provides a method to define the geometry of the three primary components, the regenerator, pulse tube and inertance tube. The sequence of topics presented here follows that given in the classroom: 1) defining the desired and required operation parameters, 2) estimating the necessary acoustic power at the warm and cold ends of the regenerator, 3) determining the dimensions of the regenerator and the mass flow and phase at its cold end, 4) fixing the pulse tube volume and dimensions, and 5) determining the length and diameter of the inertance tube.

DESIGN PROCESS

Desired and Required Parameters

From the perspective of a user, the primary parameters of interest for a cryocooler are the net cooling capacity, $\dot{Q}_c$ the desired operating temperature, T_c, and the warm – or heat rejection – temperature, T_w. Beyond these, for a Stirling-type pulse tube refrigerator, it is necessary to define the average pressure, P_o, the dynamic pressure amplitude, P_d, and the cycle frequency, f.

The choice of average pressure couples to the overall size of the system and the length scale of the regenerator matrix. If the pressure is low, a large volume will be required for effective heat transfer in the regenerator. On the other hand, high pressures will dictate optimum performance with small length scales in the regenerator matrix, and miniature size for the overall system. Practical fabrication limits for the regenerator matrix will in part dictate the possible length scales and the associated maximum average operating pressure. The time constant associated with the heat exchange process in the regenerator also dictates an inverse relationship between the characteristic length scale of the matrix and the cycle frequency. For the example developed here, a 400-mesh screen, with a porosity of 0.6858, is chosen for the matrix and the associated desired and required parameters are listed in TABLE 1. The pressure at any time during the cycle is assumed to be uniform through the refrigerator (approximately true) and given by

TABLE 1. Example values for desired and required design parameters

T_c (K)	80	$\dot{Q}_c$ (W)	25	P_o (MPa)	2.5
T_w (K)	300	f (Hz)	60	P_d (kPa)	326

$$P(t) = P_o + P_d \sin(\omega t); \quad \omega = 2\pi f \,. \tag{1}$$

The pressure ratio P_r, a parameter utilized by REGEN3.2 is related to P_o and P_d by

$$P_r = \frac{P_o + P_d}{P_o - P_d}\,. \tag{2}$$

That is, P_r is the ratio of the maximum pressure to the minimum pressure over one cycle. In this case, Pr=1.3.

Acoustic Power

The oscillating flow generated by the compressor provides acoustic power to the pulse tube refrigerator components. The magnitude of the acoustic power is related to the pressure and mass flow oscillations according to the equation:

$$\dot{W}_{ac} = \frac{1}{2} P_d \dot{m} \frac{RT}{P_o} \cos\theta \tag{3}$$

where R is the gas-specific ideal gas constant, T is the time averaged temperature and θ is the phase angle between the pressure and mass flow oscillations. It is significant to note that the acoustic power in the regenerator varies nearly linearly with the temperature, so that the acoustic power at the cold end, $\dot{W}_{ac,c}$, is approximately related to the acoustic power at the warm end by:

$$\dot{W}_{ac,c} \simeq \frac{T_c}{T_w} \dot{W}_{ac,w} \tag{4}$$

Because of the thermal and flow losses occurring in the regenerator and pulse tube, and because the acoustic power is reduced with temperature through the regenerator, the net refrigeration capacity available at the cold end of the pulse tube and regenerator is only a fraction of the acoustic power generated in the compressor. The ratio of the net cooling power to the acoustic power generated in the compressor defines the coefficient of performance for the refrigerator:

$$COP = \frac{\dot{Q}_c}{\dot{W}_{ac,w}}\,. \tag{5}$$

Thus, knowledge of the COP and the desired cooling power defines the magnitude of the required acoustic power from the compressor. Of course, it is of interest to minimize the required compressor power, and therefore one would like to know how to maximize the COP.

Mass Flow and Phase Angle at the Cold End of the Regenerator

As Radebaugh[10] points out, the dominant losses in a pulse tube refrigerator originate in the regenerator and are proportional to the average magnitude of the mass flow in that component. FIGURE 1 displays the phasor diagram relating the mass flow, pressure, and

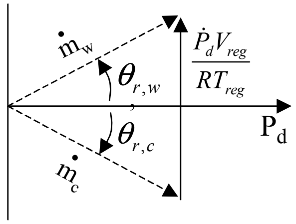

FIGURE 1. Phasor diagram displaying magnitude and phase of mass flow in the regenerator. The phase of the pressure oscillation is defined as 0°. The mass flow at the warm end of the regenerator leads the pressure by $\theta_{r,w}$ while the mass flow at the cold end lags the pressure by $\theta_{r,c}$.

mass storage terms in a regenerator. The vectoral addition reflects the equation for mass conservation in the regenerator:

$$\dot{m}_w = \dot{m}_c + \frac{\dot{P}V_{reg}}{RT_{reg}} \tag{6}$$

where the second term on the right hand side of equation (6) represents mass storage in the dead volume of the regenerator, V_{reg}, at a temperature T_{reg}, given by the reciprocal of the average of 1/T in the regenerator. Because of the sinusoidal nature of P(t), its time derivative, $\dot{P}$, and thus the second term on the right hand side of equation (6), is orthogonal to P(t). Note from equation (3) that the term $\dot{m}\cos\theta$, that is the projection of the mass flow vector onto the pressure axis, is constant throughout the regenerator. Thus adjusting the angles $\theta_{r,w}$ and $\theta_{r,c}$ away from the condition

$$\left|\theta_{r,w}\right| = \left|\theta_{r,c}\right| \tag{7}$$

results in a larger average mass flow rate in the regenerator than if equation (7) is satisfied. Although this condition is approximate, it is a key requirement for minimizing the regenerator losses and therefore maximizing the COP of the pulse tube refrigerator. Whatever phase shifting mechanisms are available in the pulse tube refrigerator should therefore be used to produce the condition given by equation (7).

From FIGURE 1 it may be noticed that with the condition defined by equation (7)

$$\dot{m}_c \sin\theta_{r,c} = \frac{1}{2}\frac{\omega P_d V_{reg}}{RT_{reg}}. \tag{8}$$

Furthermore, by combining equations (3) and (4) we have

$$\dot{m}_c \cos\theta_{r,c} = \frac{2\dot{W}_{ac,c}P_o}{P_d R T_c}. \tag{9}$$

Combining equations (8) and (9) eliminates the mass flow term, leaving

$$\theta_{r.c} = \arctan\left(\frac{\omega P_d^2 V_{reg} T_c}{4\dot{W}_{ac,c} P_o T_{reg}}\right). \tag{10}$$

Here we have also used the time derivative of equation (1) to determine the amplitude relationship:

$$\left|\dot{P}\right| = \omega P_d \tag{11}$$

Before equation (10) can be used to quantitatively determine $\theta_{r,c}$ values must be found for $\dot{W}_{ac,c}$ and V_{reg}. Knowing the COP is the key to evaluating $\dot{W}_{ac,c}$. Specifically, from equations (4) and (5), $\dot{W}_{ac,c}$ is given by

$$\dot{W}_{ac,c} = \frac{T_c}{T_w}\frac{\dot{Q}_c}{COP} \tag{12}$$

Various methods can be pursued to characterize regenerator performance and thereby determine its geometry. Recent parametric studies using REGEN3.2[5,6] provide convenient charts for this purpose. In these one finds that although the COP is influenced by many variables, optimizing over all of these reveals in the end that the maximum value of COP is most strongly influenced by T_c. For 80 K, assuming that 20% of the acoustic power flow in the pulse tube is consumed by losses there, and including conduction loss through the regenerator wall, one finds COP = 0.117. Combining this result with equations (4) and (5) yields a value of $\dot{W}_{ac,c}$ = 57.1 W. Furthermore an optimization process through the use of REGEN3.2 reveals that the oscillation frequency and end temperatures determine an optimum length of the regenerator. Also, as shown in FIGURE 2, for a fixed choice of the parameters listed in TABLE 1, a maximum COP can be identified as a function of mass flux through the regenerator. For example, with the conditions chosen in TABLE 1, REGEN3.2 finds that the COP is maximized for a regenerator length of 0.052 m and an inverse mass flux ($A_g/\dot{m}_c$) of 0.052 m^2·s/kg

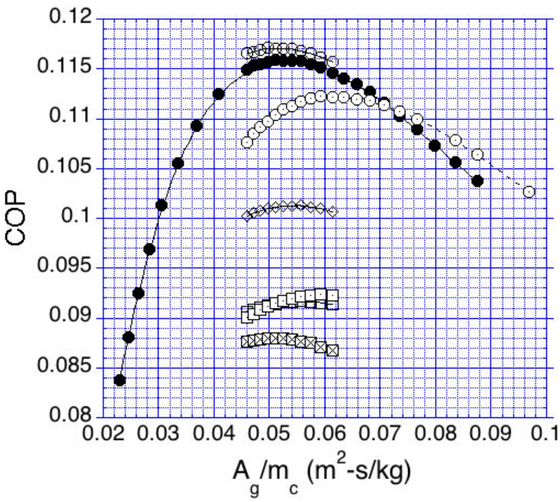

FIGURE 2. Results from parametric investigation with REGEN3.2. Solid circles: L=45 mm, P_o=2.5 MPa, P_r=1.3, $\theta_{r,c}$=-30°; open circle w/ solid line: L=52 mm, P_o=2.5 MPa, P_r=1.3, $\theta_{r,c}$=-30°; open circle w/ dashed line: L=52 mm, P_o=2.0 MPa, P_r=1.3, $\theta_{r,c}$=-30°; open diamond: L=52 mm, P_o=2.5 MPa, P_r=1.3, $\theta_{r,c}$=-10°; open square with dot: L=52 mm, P_o=2.0 MPa, P_r=1.39, $\theta_{r,c}$=0°; open square: L=52 mm, P_o=2.5 MPa, P_r=1.3, $\theta_{r,c}$=0°; open square with x: L=52 mm, P_o=3.0 MPa, P_r=1.24, $\theta_{r,c}$=0°.

respectively. Here $A_g = V_{reg}/L_{reg}$ is the cross sectional area of gas flow; equal to the total cross sectional area times the porosity.

As long as the optimized mass flux is held constant, the cooling capacity of the regenerator can be scaled directly with the regenerator area. This fact can also be appreciated by inspection of the linear relationship between the amplitude of the mass flow at the cold end of the regenerator and the cooling capacity. The combination of equations (4), (5) and (9) provides the relationship

$$\dot{m}_c = \frac{2}{T_w}\frac{\dot{Q}_c}{COP}\frac{P_o}{P_d}\frac{1}{R\cos\theta_c} \quad . \tag{13}$$

Combining equation (13) with the optimum inverse mass flux provides an expression for the gas flow area:

$$A_g = \left(\frac{A_g}{\dot{m}_c}\right)_{opt}\frac{2}{T_w}\frac{\dot{Q}_c}{COP}\frac{P_o}{P_d}\frac{1}{R\cos\theta_c} = \frac{V_{reg}}{L_{reg}} \, . \tag{14}$$

An iterative solution to equations (10) and (14) yields the values of A_g and $\theta_{r,c}$. For the given example, $A_g = 3.54 \times 10^{-4}$ m^2 (with associated regenerator diameter of 25.6 mm) and $\theta_{r,c} = -39°$. Additionally, the amplitude of the mass flow at the cold end is 6.81 g/s. Note that the compressor must provide the same values of mass flow and phase (with opposite sign) at the warm end of the regenerator. Having defined the optimum geometry for the regenerator, the design process moves next to the pulse tube.

Pulse Tube Volume and Dimensions

The gas flow oscillation from the cold end of the regenerator into the pulse tube defines a cold end swept volume, $V_{c,pt}$ the maximum value of which is given by

$$V_{c,pt} = \frac{2\dot{V}_c}{\omega} = \frac{2\dot{m}_c R T_c}{\omega P_o} \, . \tag{15}$$

Based on an empirical 'rule of thumb'[10], the total volume of the pulse tube should be three to five times larger than $V_{c,pt}$. If the pulse tube is designed for low temperature operation (4 K - 20 K) the value of five times $V_{c,pt}$ is appropriate, while for operation around 100 K, three times $V_{c,pt}$ is the better choice. In either case, if the pulse tube is too small, excessive losses are caused by penetration of the cold oscillation to the warm end. Alternately, if the volume is too large, excessive power is required from the compressor to develop the desired dynamic pressure amplitude. Continuing with the example calculation, a value of 3.5 for operation at 80 K is selected and results in a pulse tube volume of 8400 mm^3.

A limit on the pulse tube aspect ratio may be identified by considering the conditions necessary to avoid turbulence in the oscillating boundary layer at the pulse tube walls. As shown by Akhavan *etal.*[11] the critical Reynolds number for oscillating flow is given by

$$\mathrm{Re}_{crit} = \frac{\rho u_{\mathrm{crit}}\delta}{\mu} = 280; \quad \delta = \sqrt{\frac{2\nu}{\omega}} \tag{16}$$

where ρ, u, and μ are the density, amplitude of the cross-sectional mean velocity, and dynamic viscosity respectively, while δ is the boundary layer thickness dependent on ν, the kinematic viscosity and ω, the angular frequency. The maximum limit of the cross-sectional mean velocity defined by equation (16) imposes a minimum limit to the cross sectional area of the pulse tube for a given mass flow rate. From equation (15) it can be seen that a maximum velocity and the associated minimum area are related by

$$u_{crit} A_{\min} = \dot{V} = \frac{\dot{m}RT}{P_o} . \tag{17}$$

From equations (16) and (17) then, the limit on the cross-sectional area is:

$$A_{\min} = \frac{\dot{m}\delta}{\mathrm{Re}_{crit}\,\mu} . \tag{18}$$

The limit of cross-sectional area must be considered at both the warm and cold ends. The larger minimum area will then define the minimum cross sectional area for the pulse tube. In most cases, the cold end conditions will define the limit. However, to calculate equation (18) for the warm end, a value of $\dot{m}_{pt,w}$ must be found. From the equation for conservation of energy in the pulse tube[4] the vectoral diagram displayed in FIGURE 3 results. Here γ is the specific heat ratio, c_p / c_v. An expression for the magnitude of the mass flow at the warm end of the pulse tube is then obtained geometrically from FIGURE 3:

$$\dot{m}_{pt,w} = \frac{T_c}{T_h}\left\{\left(\frac{2\dot{W}_{ac,c}P_o}{P_d R T_c}\right)^2 + \left[\left(\frac{\omega P_d V_{reg}}{2RT_{reg}}\right) + \left(\frac{\omega P_d V_{pt}}{\gamma R T_c}\right)\right]^2\right\}^{1/2} . \tag{19}$$

For the example calculation, $A_{c,min} = 1.57\text{x}10^{-4}\text{m}^2$ and $A_{w,min} = 7.49\text{x}10^{-5}\text{m}^2$. Using $A_{c,min}$, the diameter and length of the pulse tube become d_{pt} = 14.1 mm and L_{pt} = 53.6 mm. Finally, utilizing the geometric relations associated with θ_{pt} displayed in FIGURE 3,

$$\begin{aligned} \dot{m}_{pt,w}\left(\frac{T_w}{T_c}\right)\cos\theta_{pt} &= \dot{m}_c \cos\theta_{r,c} = \frac{2\dot{W}_{ac,c}P_o}{P_d R T_c} \\ \dot{m}_{pt,w}\left(\frac{T_w}{T_c}\right)\sin\theta_{pt} &= \frac{\omega P_d V_{reg}}{2RT_{reg}} + \frac{\omega P_d V_{pt}}{\gamma R T_c} \end{aligned} \tag{20}$$

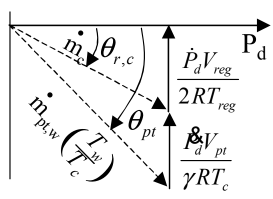

FIGURE 3. Phase diagram displaying the relationship between the mass flow vectors in the pulse tube as defined by the conservation of energy in the pulse tube.

an expression for θ_{pt} is obtained:

$$\theta_{pt} = \arctan\left[\frac{\omega P_d^2\left(V_{reg}\gamma T_c + 2V_{pt}T_{reg}\right)}{4\gamma T_{reg}\dot{W}_{ac,c}P_o}\right] \tag{21}$$

For the example parameters defined above, $\theta_{pt} = 57°$.

Inertance tube dimensions

The diameter and length of the inertance tube are determined by the required phase angle, θ_{pt}, where it interfaces with the warm end of the pulse tube, and the acoustic power flow at the same location. Various inertance tube models are available[7,8] and show that for smaller values of acoustic power (less than 1 watt), the inertance tube does not provide any appreciable phase shift.. For the example calculation carried out here, the model by Schunk *etal.*[7] finds that for P_o = 2.5 MPa, P_r = 1.3, $\dot{W}_{ac}$ = 57.1 watts and θ = 57°, the required length and diameter for the inertance tube are 2.92 m and 4.67 mm respectively.

SUMMARY

A method for approximating the design of a pulse tube refrigerator has been presented. For user defined parameters of cold end temperature and cooling power, the method requires definition of the average and dynamic pressure and cycle frequency. Utilizing established design charts for the regenerator and inertance tube, and phasor diagrams, the approach provides a method to define the diameter and length for the pulse tube, regenerator, and inertance tube components as well as the required performance of the compressor including acoustic power, mass flow, and pressure-flow phase relationship.

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