Published in:
Advances in Cryogenic Engineering
vol. 53, American Institute of Physics,
(2008), pp. 225-234.

CALCULATED REGENERATOR PERFORMANCE AT 4 K WITH HELIUM-4 AND HELIUM-3*

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ABSTRACT

The helium-4 working fluid in regenerative cryocoolers operating with the cold end near 4 K deviates considerably from an ideal gas. As a result, losses in the regenerator, given by the time-averaged enthalpy flux, are increased and are strong functions of the operating pressure and temperature. Helium-3, with its lower boiling point, behaves somewhat closer to an ideal gas in this low temperature range and can reduce the losses in 4 K regenerators. An analytical model is used to find the fluid properties that strongly influence the regenerator losses as well as the gross refrigeration power. The thermodynamic and transport properties of helium-3 were incorporated into the latest NIST regenerator numerical model, known as REGEN3.3, which was used to model regenerator performance with either helium-4 or helium-3. With this model we show how the use of helium-3 in place of helium-4 can improve the performance of 4 K regenerative cryocoolers. The effects of operating pressure, warm-end temperature, and frequency on regenerators with helium-4 and helium-3 are investigated and compared. The results are used to find optimum operating conditions. The frequency range investigated varies from 1 Hz to 30 Hz, with particular emphasis on higher frequencies.

KEYWORDS: Cryocoolers, cryogenics, Gifford-McMahon, helium-3, helium-4, numerical analysis, pulse tubes, real gas, refrigeration, regenerators, Stirling, theory

INTRODUCTION

The application of low temperature superconducting (LTS) systems, such as magnetic resonance imaging (MRI) systems utilizing superconducting magnets or electronic devices utilizing Josephson junctions, requires the use of 4 K cryocoolers. Typically these cryocoolers have been either Gifford-McMahon (GM) cryocoolers or GM-type pulse tube cryocoolers that operate at frequencies of about 1 Hz [1]. The efficiency of these cryocoolers is in the range of 0.5 to 1.0 % of Carnot, whereas 80 K cryocoolers often

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achieve efficiencies of at least 15 % of Carnot. The low efficiency of 4 K cryocoolers leads to large compressors with large input powers. The low operating frequency of the GM and GM-type pulse tube also leads to large temperature oscillations at the cold end at the operating frequency of the cryocooler. The amplitude of the temperature oscillation decreases as the cryocooler operating frequency is increased. Higher frequencies also allow the use of Stirling cryocoolers or Stirling-type pulse tube cryocoolers, which have much higher efficiencies in converting electrical power to PV power. These frequencies are typically in the range of 30 to 60 Hz. However, these higher frequencies generally lead to greater losses in the regenerator. Recent work with a 4 K GM-type pulse tube [2,3] and a Stirling-type pulse tube cryocooler [4] has shown that the use of ^{3}He instead of ^{4}He increased the cooling power for the same power input. A systematic comparison of regenerator behavior with ^{3}He and ^{4}He has not been carried out previously. Such a comparison is the subject of this paper.

HELIUM-3 PROPERTIES

The Debye equation used to express the temperature dependence of the specific heat of solids was used by Huang *et al.* [5,6] to fit published experimental data for the thermodynamic properties of ^{3}He for temperatures from 0.01 K to 1500 K and pressures up to 20 MPa. Deviations between this Debye equation of state and the reference experimental data were within ±1 %. A graphical computer program “He3Pak” was developed for the calculation of ^{3}He thermodynamic properties over this range of temperatures and pressures. However, there are no experimental data for the transport properties of He^3 in the gas phase. Thus, a quantum version of the principle of corresponding states was used to calculate the viscosity, thermal conductivity, and surface tension of ^{3}He in the gas phase. This quantum version uses the intermolecular potential constant and the molecular diameter as reduction parameters in place of the critical properties. The reduced ^{3}He transport property was found by extrapolating the corresponding reduced property for Ar, Ne, H_2, and ^{4}He on the scale for the reduced de Broglie wavelength of each of the gases.

NIST NUMERICAL MODEL REGEN3.3 WITH PROPERTIES OF HELIUM-3

The results presented in this paper were obtained with the new version 3.3 of the NIST regenerator model that includes an option to select ^{4}He or ^{3}He as the working fluid as well as the ideal gas version of either gas. The boundary conditions for the older versions, 3.1 or 3.2 [7] required the mass flow to be given at both ends of the regenerator. In the new version the mass flow and pressure are given at the cold end. This avoids an iteration to guess the mass flow at the warm end to obtain the pressure ratio and phase relative to the cold end mass flow. The latter usually determine the desired operating point for the model. Also, the new model is based on the equations for conservation of mass, momentum and energy in contrast to the old, which used derived equations. Use of the conservation equations eliminated the assumption of zero pressure gradient in the older versions (in those the gradient is estimated from the solution). In the previous versions it was necessary to correct the pressure at the end of each time step in order to insure mass conservation over thousands of cycles. This is not necessary in the new version.

The REGEN3.3 version has been compared using some cases computed earlier with the older version, REGEN3.2, and we found that the two versions gave equal or very similar results. In some cases, where geometrical properties such as area or porosity varied along the regenerator, the new version converged where the older version failed. Although the newer version requires more computer time per time-step, it may use fewer mesh points for equivalent accuracy, and the iteration for the desired pressure is no longer necessary; so we found that the new version is considerably faster. Running times with a desktop computer are typically about 20 minutes for a 4 K regenerator.

CRYOCOOLER THERMODYNAMICS

Cryocooler losses

Only the last-stage regenerator, which reaches 4 K, is considered in the analysis presented here. The time-averaged acoustic power $\langle P\dot{V}\rangle_h$ that drives this stage enters the regenerator at the hot end at a temperature of T_h. It is defined by the reversible isothermal power input given by the time-averaged Gibbs free energy flow $\langle\dot{G}\rangle_h$. The purpose of the regenerator is to deliver as much of this acoustic power to the cold end as possible with a minimum of losses. If a reversible, isothermal expansion piston were placed at the cold end of the regenerator, the heat that can be absorbed at the cold end by the expansion process averaged over one cycle is given by the first law of thermodynamics for oscillating flow as

$$\langle\dot{Q}\rangle_c = \langle\dot{W}\rangle_{rev} - \langle\dot{H}\rangle_{reg} - \dot{Q}_{cond}\,, \tag{1}$$

where $\langle\dot{W}\rangle_{rev}$ is the time-averaged reversible work flow produced by the expansion piston or displacer with the back side at the average pressure, $\langle\dot{H}\rangle_{reg}$ is the time-averaged enthalpy flow in the regenerator, and $\dot{Q}_{cond}$ is the heat conduction to the cold end of the regenerator. We choose the positive direction for mass and enthalpy flow to be from the hot end of the regenerator to the cold end. In any real Stirling or Gifford-McMahon cryocooler the expansion work is not reversible. A loss term can be included in equation (1) to account for irreversible expansion effects.

If the piston is replaced by a pulse tube, the acoustic power delivered to the cold end of the regenerator continues on into the pulse tube instead of being converted to the real work flow $\langle\dot{W}\rangle_{rev}$. In that case $\langle\dot{W}\rangle_{rev}$ in equation (1) must be replaced by the time averaged enthalpy flow in the pulse tube $\langle\dot{H}\rangle_{pt}$. The acoustic power at the cold end of the pulse tube is given by a combination of the first and second law of thermodynamics as [8]

$$\langle P\dot{V}\rangle_c = \langle\dot{H}\rangle_c - T_c\langle\dot{S}\rangle_c = \langle\dot{G}\rangle_c\,, \tag{2}$$

where P is the dynamic pressure, $\dot{V}$ is the volume flow rate, $\dot{H}$ is the enthalpy flow, T_c is the cold temperature, $\dot{S}$ is the entropy flow, and $\dot{G}$ is the Gibbs free energy flow. The enthalpy flow anywhere within the pulse tube is then given by

$$\langle\dot{H}\rangle = \langle P\dot{V}\rangle + T\langle\dot{S}\rangle. \tag{3}$$

Within a perfectly adiabatic pulse tube $\langle \dot{S} \rangle = 0$, so $\langle \dot{H} \rangle_c = \langle P\dot{V} \rangle_c$. Irreversible processes in any real pulse tube generate entropy that flows toward the compressor ($\langle \dot{S} \rangle < 0$). A refrigeration degradation factor or a loss $\dot{Q}_{pt}$ can be used to account for such losses.

The loss associated with the enthalpy flow $\langle \dot{H} \rangle_{reg}$ in the regenerator can be divided into two parts, as given by

$$\langle \dot{H} \rangle_{reg} = \langle \dot{H} \rangle_P + \dot{Q}_{reg}, \tag{4}$$

where $\langle \dot{H} \rangle_P$ is the enthalpy flow associated with the enthalpy pressure dependence (real gas effects) and $\dot{Q}_{reg}$ is the thermal loss associated with enthalpy flow due to imperfect heat transfer in the regenerator (regenerator ineffectiveness). This separation allows us to determine the intrinsic loss associated with using a real gas and how that differs between ^{4}He and ^{3}He. The gas properties also affect $\dot{Q}_{reg}$, but that loss depends as well on the properties of the actual regenerator. From equations (1) and (4) we can define the gross refrigeration power as that associated with a perfect regenerator and perfect expansion process and given by

$$\dot{Q}_{gross} = \langle P\dot{V} \rangle_c - \langle \dot{H} \rangle_P \,. \tag{5}$$

From equations (1) and (4) the net refrigeration power in any real system can be given by

$$\dot{Q}_{net} = \langle P\dot{V} \rangle_c - \langle \dot{H} \rangle_P - \dot{Q}_{reg} - \dot{Q}_{cond} - \dot{Q}_{pt}, \tag{6}$$

where $\dot{Q}_{pt}$ is the loss associated with an imperfect pulse tube or any irreversible expansion process at the cold end.

The acoustic power anywhere along the regenerator with perfect heat transfer and no pressure drop varies as the specific volume. In the presence of a pressure drop the cold-end acoustic power is related to the hot-end acoustic power by

$$\langle P\dot{V} \rangle_c = \left(Z_c T_c / Z_h T_h \right) \left[\langle P\dot{V} \rangle_h - \langle \Delta P\dot{V} \rangle_h \right], \tag{7}$$

where Z_c is the compressibility factor at the cold end, Z_h is the compressibility factor at the hot end, and $\langle \Delta P\dot{V} \rangle_h$ is the acoustic power lost at the hot end due to pressure drop. By combining equations (5), (6), and (7) we can express the net refrigeration power as

$$\dot{Q}_{net} = \langle P\dot{V} \rangle_h \left[1 - \frac{\langle \Delta P\dot{V} \rangle_h}{\langle P\dot{V} \rangle_h} \right] \left[\frac{Z_c T_c}{Z_h T_h} \right] \left[1 - \frac{\langle \dot{H} \rangle_P}{\langle P\dot{V} \rangle_c} \right] \left[1 - \frac{\dot{Q}_{reg}}{\dot{Q}_{gross}} - \frac{\dot{Q}_{cond}}{\dot{Q}_{gross}} - \frac{\dot{Q}_{pt}}{\dot{Q}_{gross}} \right]. \tag{8}$$

By writing the net refrigeration power in this manner, we have separated out the terms that are functions only of the gas properties from those that depend also on the hardware. The first term on the right hand side of the equation is the acoustic power input at the hot end of the regenerator. The second term shows the effect of pressure drop in the regenerator and is both hardware and gas dependent. The third term shows the reduction in acoustic power due to temperature change and real-gas behavior associated with compressibility. The fourth term shows the effect of real-gas enthalpy flow. The terms in the last set of brackets are both hardware and gas dependent.

FIGURE 1 shows a schematic of the energy flows and losses associated with the last stage of a regenerative cryocooler as represented by equation (8). The relative magnitudes shown for each of the acoustic power flows and the losses are typical of a regenerative cryocooler at 4 K. As this figure shows, the losses are quite large and the remaining net refrigeration power is quite small compared to the input power.

Coefficient of performance and efficiency

The coefficient of performance of the last stage regenerator is given by

$$\mathrm{COP} = \frac{\dot{Q}_{net}}{\langle P\dot{V}\rangle_h}. \tag{9}$$

For an ideal gas and a perfect regenerator the ideal COP for this last stage regenerator is given by (T_c/T_h), where we assume that the reversible expansion work at the cold end is not being fed back to the hot end of this regenerator. Thus, the second law efficiency of the last stage is given by

$$\eta = \left(T_h/T_c\right)\mathrm{COP}. \tag{10}$$

Combining equations (8), (9), and (10) gives the second law efficiency of the last stage as

$$\eta = \left[1 - \frac{\langle \Delta P\dot{V}\rangle_h}{\langle P\dot{V}\rangle_h}\right]\left[\frac{Z_c}{Z_h}\right]\left[1 - \frac{\langle \dot{H}\rangle_P}{\langle P\dot{V}\rangle_c}\right]\left[1 - \frac{\dot{Q}_{reg}}{\dot{Q}_{gross}} - \frac{\dot{Q}_{cond}}{\dot{Q}_{gross}} - \frac{\dot{Q}_{pt}}{\dot{Q}_{gross}}\right]. \tag{11}$$

REAL GAS EFFECTS

When only the real gas effects are taken into account the net refrigeration power equals the gross refrigeration power, as given by equation (5). For a perfect regenerator the lost acoustic power in equation (7) is zero. The efficiency for the last stage becomes

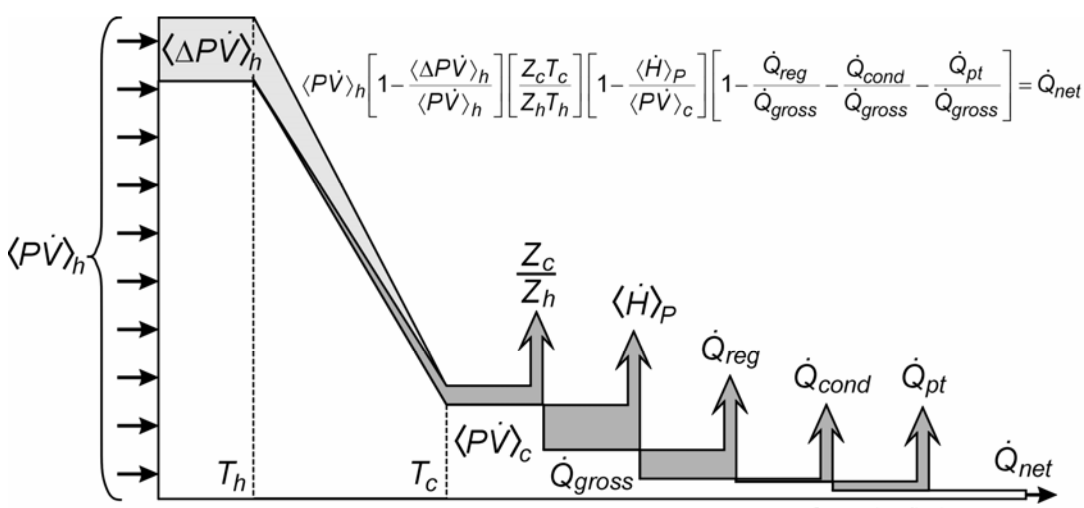

FIGURE 1. Diagram showing energy flows and losses in a regenerator.

$$\eta_{gross} = \frac{Z_c}{Z_h}\left(1 - \frac{\langle \dot{H} \rangle_P}{\langle P\dot{V} \rangle_c}\right). \tag{12}$$

The enthalpy flow associated with the real gas effects can be found by using a first law energy balance on the regenerator with perfect isothermal heat exchangers on each end along with the condition that the hot-blow stream must be warmer than the cold-blow stream. For the case where the conduction is small, this enthalpy flow is then given by

$$\langle \dot{H} \rangle_P = \max\left\{ \left\lfloor (1/\tau)\oint \dot{m}(h)_T\, dt \right\rfloor_c, \left\lfloor (1/\tau)\oint \dot{m}(h)_T\, dt \right\rfloor_h \right\}, \tag{13}$$

where τ is the period, $\dot{m}$ is the mass flow rate, h is the specific enthalpy, and t is time. For small pressure amplitudes ($P/P_0 < 0.2$ or $P_r < 1.5$, where P is the dynamic pressure, P_0 is the average pressure, and P_r is the pressure ratio) the enthalpy can be expressed as

$$h = h_0 + dh, \tag{14}$$

where h_0 is the specific enthalpy at the average pressure, and dh is the small change in enthalpy due to the oscillating pressure dP. The enthalpy flow can then be given as

$$(1/\tau)\oint \dot{m}(h)_T\, dt = \oint \dot{m} dh = \rho_0\left(\frac{\partial h}{\partial P}\right)_T \oint \dot{V} dP = \rho_0\left(\frac{\partial h}{\partial P}\right)_T \langle P\dot{V} \rangle. \tag{15}$$

The enthalpy flow relative to the acoustic power at the cold end is then given as

$$\frac{\langle \dot{H} \rangle_P}{\langle P\dot{V} \rangle_c} = \max\left\{ \left[\rho\left(\frac{\partial h}{\partial P}\right)_T\right]_c, \left[\rho\left(\frac{\partial h}{\partial P}\right)_T\right]_h \left(\frac{Z_h T_h}{Z_c T_c}\right) \right\}, \tag{16}$$

where the effect of the pressure drop has been neglected for the term at the high temperature end (second term). Use of thermodynamic identities yields

$$\rho\left(\frac{\partial h}{\partial P}\right)_T = 1 - T\beta, \tag{17}$$

where $\beta = (1/v)(\partial v/\partial T)_P$ is the volume expansivity.

FIGURE 2 shows the temperature dependence of $\rho(\partial h/\partial P)_T$ for ^{4}He and ^{3}He at various pressures. For 4 K regenerators, the maximum term occurs at the cold end, except for very low pressures. Note that for pressures less than about 1.5 MPa the real gas enthalpy flow for ^{3}He is less than that of ^{4}He. Low pressures are desirable for both gases to reduce this enthalpy flow. FIGURE 3 shows the efficiency factor $\left[1 - \langle \dot{H} \rangle_P / \langle P\dot{V} \rangle_c\right]$ associated with the enthalpy flow. Although the curves show fairly sharp peaks, the actual behavior if a finer plotting interval is used is an abrupt change in slope where the enthalpy flow changes between the two terms in equation (16).

FIGURE 4 compares the compressibility of ^{4}He and ^{3}He. Because the compressibility of both gases is close to 1 at 20 K, this graph gives a close approximation to the behavior of Z_c/Z_h. Higher values are desirable for this ratio, which occur at higher pressures.

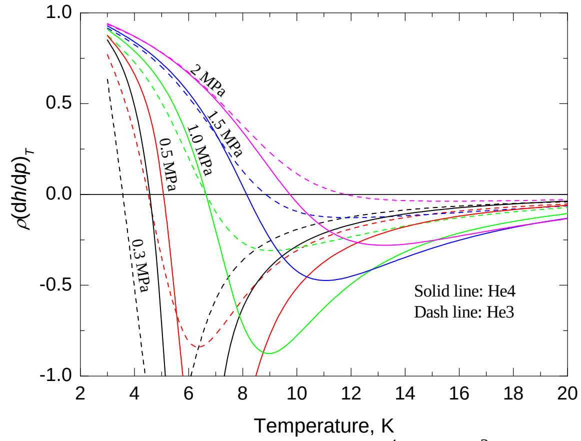

FIGURE 2. Pressure derivative of ^{4}He and ^{3}He given by equation (17) and equal to the relative enthalpy flow due to real gas effects according to equation (16).

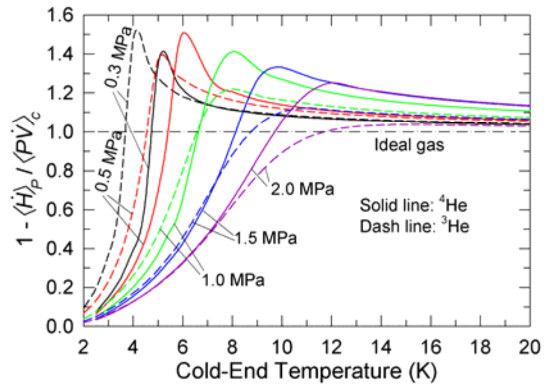

FIGURE 3. Contribution of real-gas enthalpy flow to the efficiency of a stage with the hot end at 20 K.

FIGURE 5 shows the overall efficiency η_{gross} from equation (12), which is the second law efficiency of the last stage using real gas in a perfect regenerator and pulse tube (or displacer). It is also the ratio of the real gas COP to the ideal gas COP for a perfect regenerator and pulse tube. Although these curves are for a hot-end temperature of 20 K, there is little change for higher hot-end temperatures. For 4 K cold-end temperatures, lower average pressures yield higher efficiencies. For cold-end temperatures somewhat higher than 4 K the enthalpy flow is negative (flow from cold end to warm end of regenerator) as determined by the hot end enthalpy flow. In this case the real gas COP and efficiency can actually be higher than that of the ideal gas, but it is compensated by the fact that the enthalpy flow toward the hot end causes more heat to be rejected at that temperature, which must be absorbed by the stage above this last stage.

CALCULATED REGENERATOR LOSSES WITH HELIUM-4 AND HELIUM-3

Regenerator details

We found previously [9] that the ratio of the regenerator gas volume to the cold-end swept volume is a fundamental parameter that affects the performance of 4 K regenerators operating with ^{4}He working fluid. Because both the conduction and pressure drop are rather small for most 4 K regenerators, the aspect ratio has little influence on the performance, although the regenerators modeled here are close to optimum in aspect ratio.

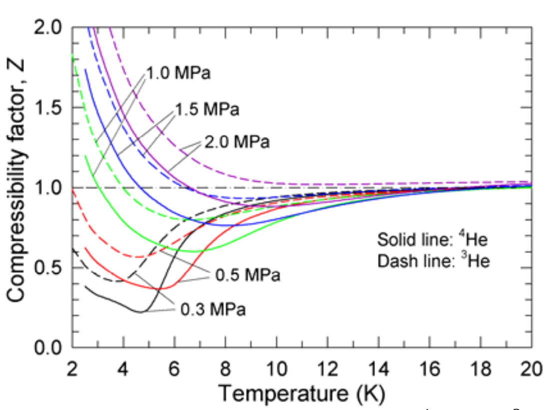

FIGURE 4. Compressibility factor for ^{4}He and ^{3}He.

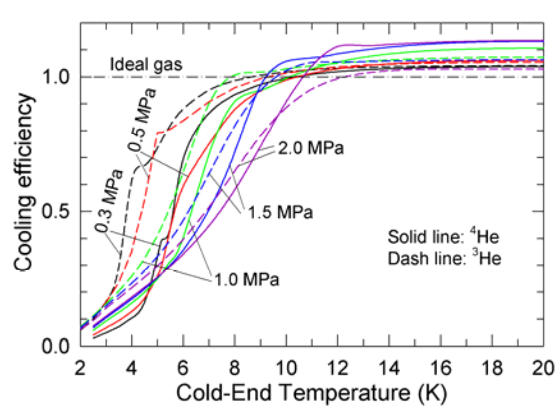

FIGURE 5. Ratio of real gas COP to ideal gas COP for last stage of a perfect regenerative cryocooler.

TABLE 1. Geometry, material, and operating conditions for regenerator modeled here.

Regenerator	**1 Hz**	**30 Hz**
Diameter (mm)	27.8	12.4
Length (mm)	180	30
Sphere diameter (μm)	250	100
Porosity	0.38	0.38
Material	Mix 1	Mix 1
Conduction factor	0.3	0.3

Conditions	
Cold temperature (K)	4.0
Hot temperature (K)	20
Average pressure (MPa)	1.0
Pressure ratio	1.5
Phase (cold flow – press.)	-30°

TABLE 1 lists the important parameters used for the regenerator and the operating conditions in most of the runs with REGEN3.3. In some cases these parameters were varied from the values given in the table to determine the effect of these parameters. The mass flow rate was varied from 1 to 5 g/s for ^{4}He and 0.75 to 3.75 g/s for ^{3}He for many runs to find the dependence on the flow rate. Spheres of 38 % porosity were chosen as the matrix material because they are commercially available. However, we have shown previously [9] that low porosity significantly reduces the regenerator loss. The material chosen consists of layers we refer to as Mix 1. The properties of this layered mixture are built into REGEN3.3. FIGURE 6 shows the composition and volumetric heat capacity of these layers compared with the heat capacity of ^{4}He, ^{3}He, and an ideal gas. Also shown in the figure is the heat capacity of the regenerator material GOS [10], which could reduce the regenerator loss if it were added to the mix. The output of REGEN3.3 will give the temperature profile to determine the location of each layer. The lower heat capacity of ^{3}He should lead to a lower regenerator loss compared with ^{4}He.

Calculated losses from REGEN3.3

FIGURE 7 compares the calculated relative regenerator loss for ^{4}He and ^{3}He as a function of the reduced volume ratio, where V_{rg} is the regenerator gas volume and V_E is the swept volume of the expansion space at the cold end. As expected, the loss with ^{3}He is less than that with ^{4}He. Large regenerator volumes lead to a reduced loss, but for volume ratios of about 10 or higher the phase of the mass flow at the warm end compared with the pressure (shown in FIGURE 7 by each data point) becomes rather high and leads to a large compressor swept volume and higher losses in the warmer regenerators needed for precooling. Though not shown here, we found that varying the average pressures between 0.5 MPa and 1.5 MPa has little effect on the regenerator loss. As shown by equation (8) the net refrigeration power becomes zero when $\dot{Q}_{reg} / \dot{Q}_{gross} = 1$, as long as there is no conduction or pulse tube loss. In the cases analyzed here, the conduction loss is negligible.

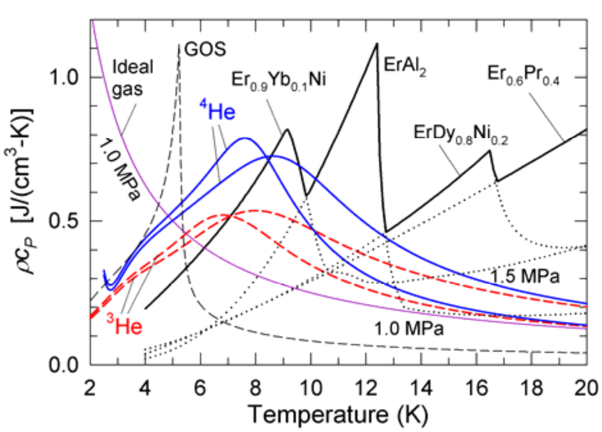

FIGURE 6. Volumetric heat capacity of regenerator material Mix 1(heavy line) and its components (dotted lines) compared with ^{4}He, ^{3}He, and an ideal gas. The material GOS was not included in Mix 1.

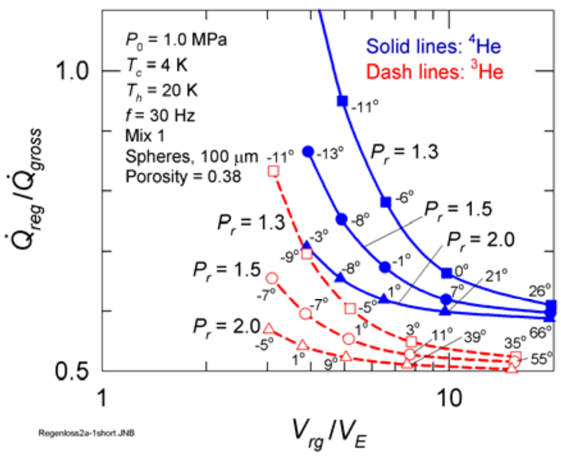

FIGURE 7. Reduced regenerator loss at 30 Hz.

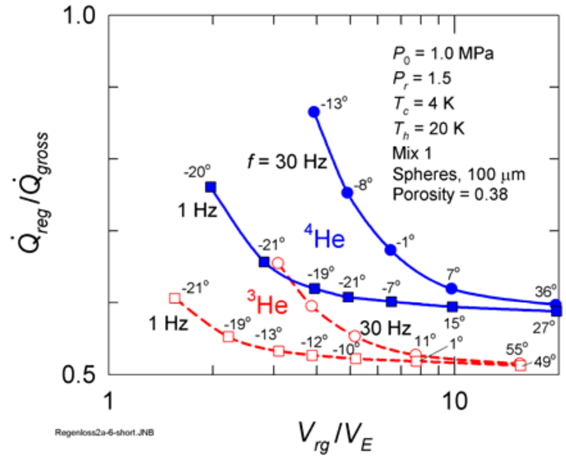

FIGURE 8. Frequency effect on regenerator loss.

However, the relative pulse tube loss in an actual cryocooler may be as large as 0.3, which means that the relative regenerator loss must be less than 0.7 to provide any net refrigeration power. The relative pressure drop at the hot end for volume ratios in the range of 7 to 10 is about 4 % to 5 % of the input acoustic power.

FIGURE 8 shows the effect of frequency on the regenerator loss for an average pressure of 1.0 MPa and a pressure ratio of 1.5. The surprising result here is that the loss does not increase much with frequency for ^{3}He as long as the volume ratio is near the optimum value ($5 < V_{rg}/V_E < 10$). However, at 1 Hz it is possible to use smaller volume ratios and a lower hot-end phase angle before the loss begins to increase significantly. FIGURE 9 shows the effect of the hot-end temperature on the regenerator loss for the case of a fixed volume ratio. We note that this temperature has negligible effect on the loss for all temperatures below 30 K for ^{4}He and below 40 K for ^{3}He. Such a result is an important design consideration.

Calculated coefficient of performance and efficiency

FIGURE 10 shows the effect of the hot-end temperature on the product (T_h/T_c)COP, which is the efficiency according to equation (10) and the ratio of the actual COP to the COP using ideal gas and a perfect regenerator and pulse tube. The values given in FIGURE 10 are those calculated with REGEN3.3 assuming no expansion loss but with the calculated pressure drop in the regenerator. For comparison we note from FIGURE 5 that the estimated real gas efficiency at 4 K for an average pressure of 1.0 MPa is 26 % for ^{3}He and 16 % for ^{4}He. When these efficiencies are multiplied by the regenerator factor

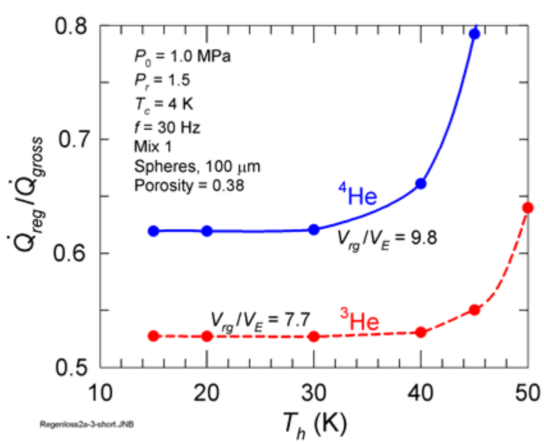

FIGURE 9. Effect of hot-end temperature on regenerator loss for optimum volume ratio.

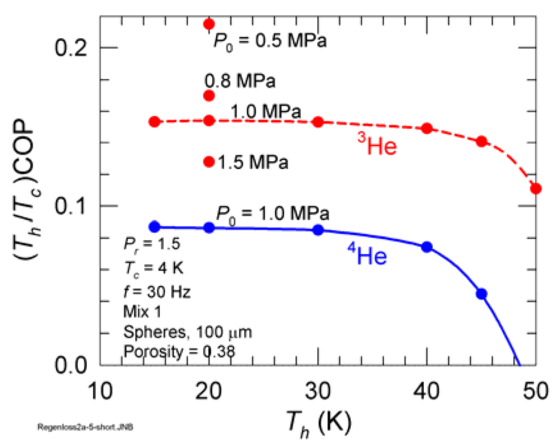

FIGURE 10. Effect of hot-end temperature on last-stage second-law efficiency.

$1-(\dot{Q}_{reg}/\dot{Q}_{gross})$, the result is an overall efficiency of 12.3 % for ^{3}He and 6.1 % for ^{4}He, compared with the values of 15.4 % and 8.7 % calculated directly with REGEN3.3 for an actual system, as shown in FIGURE 10. We emphasize that the efficiencies given here are only for the last stage and not for the complete system. Also shown in FIGURE 10 are the efficiencies for ^{3}He at average pressures of 0.5, 0.8, and 1.5 MPa for a 20 K hot-end temperature. Higher efficiencies are observed at the lower pressures, as expected from the curves in FIGURE 5.

CONCLUSIONS

We have shown that the thermodynamic properties of ^{3}He are such that when it is used as the working fluid in a perfect regenerative cryocooler, the efficiency will be higher than one using ^{4}He as long as the average pressure is less than about 1.5 MPa and the cold-end temperature is less than about 10 K. For 4 K operation lower average pressures lead to higher real gas efficiencies. The thermodynamic and transport properties have been incorporated into a new numerical model called REGEN3.3 that was used to calculate the performance of actual regenerative cryocooelers using either ^{3}He or ^{4}He as the working fluid. The results show that the regenerator loss at 4 K is reduced by using ^{3}He. For optimally sized regenerators the loss is nearly independent of frequency between 1 Hz and 30 Hz. Hot-end temperatures up to about 40 K can be used before any significant increase in regenerator loss is observed. The lower regenerator loss experienced when using ^{3}He is explained by the lower volumetric heat capacity of ^{3}He compared with that of ^{4}He.

ACKNOWLEDGMENT

This research was partially funded by the Office of Naval Research with Deborah Van Vechten as the project monitor.

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