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REGENERATOR BEHAVIOR AT 4 K: EFFECT OF VOLUME AND POROSITY*

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ABSTRACT

The low heat capacity of regenerator materials near 4 K and the real gas properties of helium at this temperature give rise to anomalous temperature profiles and behavior not seen at higher temperatures. This paper describes the behavior of regenerators calculated from the NIST computer code REGEN3.2 when the cold end is at 4 K. The results show that the regenerator loss is independent of the regenerator volume over a wide range and does not increase until the volume is decreased below some critical value, at which point the loss increases very rapidly. The transition occurs when the gas displacement amplitude at the warm end approaches the length of the regenerator. The model shows that reducing the porosity leads to a decreased regenerator loss at the plateau. The paper also describes the effect of the temperature at the warm end and the matrix heat capacity on the regenerator loss. The calculated temperature profiles agree with experimental measurements on regenerators in 4 K Gifford-McMahon refrigerators.

INTRODUCTION

Small cryocoolers for temperatures of 4.2 K have efficiencies of only about 1% of Carnot. A refrigeration power of 0.25 W at 4.2 K is a typical requirement for the cooling of superconducting electronics, but the input power of 2.5 kW required to drive such a system often makes it inconvenient for such applications. These small cryocoolers are either Gifford-McMahon (GM) cryocoolers or pulse tube cryocoolers driven with GM compressors. Much of the cryocooler inefficiency is associated with these valved compressors. The isothermal PV power associated with the steady or DC pressures generated by these compressors is only about 50% of the electrical input power. The valves used to convert this steady pressure into an oscillating pressure of about 1 to 2 Hz frequency reduce the overall compression efficiency to something less than 40%. Valveless pressure oscillators, or Stirling-type compressors, usually have efficiencies around 70 to 80%, but frequencies of at least 20 Hz are required to keep the size of the compressor from being too large. However, the behavior of 4 K regenerators at such high frequencies is not well understood. The regenerator losses appear to increase with frequency and become so large as to prevent the use of the efficient Stirling-type compressors with 4 K cryocoolers. The purpose of this paper is to discuss some results of modeling 4 K regenerators that may lead to reducing their losses and allow their use at frequencies of 20 Hz or higher. These results can also lead to improved cryocooler efficiencies at low frequencies.

THERMODYNAMICS OF REGENERATORS

Regenerators function as heat exchangers by storing heat in the matrix for a half cycle. To maintain a high effectiveness the amplitude of the matrix temperature oscillation should be as small as possible, that is, the amount of heat transferred to the matrix should be minimized and the matrix specific heat should be as high as possible. Because matrix specific heats are quite low at 4 K, it is especially important to minimize the amount of heat transferred between the gas and the matrix. The heat transferred per unit gas volume to the gas from the matrix at any location in the regenerator is given from a first law energy balance as

$$\dot{q}_{vg} = \frac{d\dot{Q}}{A_g dx} = \frac{\partial}{\partial x}\left[\left(\frac{\dot{m}}{A_g}\right)h\right] - \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial(\rho u)}{\partial t}, \tag{1}$$

where $\dot{m}$ is the mass flow rate, h is the specific enthalpy, A_g is the gas cross-sectional area, x is the position along the flow axis with positive x being toward the cold end, k is the thermal conductivity of the gas, ρ is the density, u is the specific internal energy, and t is the time. The first term on the right hand side is the heat flow caused by mass flow across a temperature gradient, the second term is the heat flow caused by the gradient in heat conduction in the gas, and the last term is the heat flow caused by compression and expansion of the gas in the void space. The term due to the gradient in gas conduction is usually negligible, especially for temperatures around 4 K. By ignoring the conduction term and expanding the others we obtain

$$\dot{q}_{vg} = h\frac{\partial}{\partial x}\left(\frac{\dot{m}}{A_g}\right) + \left(\frac{\dot{m}}{A_g}\right)\frac{\partial h}{\partial x} + \rho\frac{\partial u}{\partial t} + u\frac{\partial \rho}{\partial t}. \tag{2}$$

We then make the substitution

$$u = h - (P/\rho), \tag{3}$$

where P is the pressure, to get

$$\dot{q}_{vg} = h\frac{\partial}{\partial x}\left(\frac{\dot{m}}{A_g}\right) + \left(\frac{\dot{m}}{A_g}\right)\frac{\partial h}{\partial x} + \rho\frac{\partial u}{\partial t} + h\frac{\partial \rho}{\partial t} - \frac{P}{\rho}\frac{\partial \rho}{\partial t}. \tag{4}$$

For the first term on the right hand side of equation (4) we use the conservation of mass equation

$$\frac{\partial}{\partial x}\left(\frac{\dot{m}}{A_g}\right)=-\frac{\partial \rho}{\partial t} \tag{5}$$

to obtain

$$\dot{q}_{vg}=\left(\frac{\dot{m}}{A_g}\right)\frac{\partial h}{\partial x}+\rho\frac{\partial u}{\partial t}-\frac{P}{\rho}\frac{\partial \rho}{\partial t}. \tag{6}$$

The change in the specific internal energy can be expressed as

$$du=\left(\frac{\partial u}{\partial T}\right)_P dT+\left(\frac{\partial u}{\partial P}\right)_T dP. \tag{7}$$

Substituting in thermodynamic identities into equation (7) gives

$$du=c_v dT+v(\kappa P-\beta T)dP, \tag{8}$$

where κ is the isothermal compressibility and β is the volume expansivity. For an ideal gas u is independent of P so the second term on the right hand side of equations (7) and (8) is zero. In a similar manner the change in density can be expressed as

$$\begin{aligned}\frac{d\rho}{\rho}&=\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P dT+\frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T dP\\ &=-\beta dT+\kappa dP.\end{aligned} \tag{9}$$

Substituting equations (8) and (9) into equation (6) yields

$$\dot{q}_{vg}=c_p\left(\frac{\dot{m}}{A_g}\right)\frac{\partial T}{\partial x}+(\rho c_v+\beta P)\frac{\partial T}{\partial t}-\beta T\frac{\partial P}{\partial t}, \tag{10}$$

where the enthalpy change was replaced with $dh = c_p dT$.

The heat flow from the gas to the matrix per unit gas volume is given as

$$-\dot{q}_{vg}=\left[\frac{1-n_g}{n_g}\right]\rho_m c_m\frac{dT_m}{dt}, \tag{11}$$

when matrix conduction is ignored, and where n_g is the porosity and the subscript m refers to the matrix. By combining equations (10) and (11) we obtain

$$\left[\frac{1-n_g}{n_g}\right]\rho_m c_m\frac{dT_m}{dt}=-c_p\left(\frac{\dot{m}}{A_g}\right)\frac{\partial T}{\partial x}-(\rho c_v+\beta P)\frac{\partial T}{\partial t}+\beta T\frac{\partial P}{\partial t}. \tag{12}$$

At temperatures around 4 K the regenerator performance is limited by the low volumetric heat capacity of the matrix material. Thus, we try to reduce the heat flow to the matrix per unit matrix volume, as given by

$$\dot{q}_{vm} = \rho_m c_m \frac{dT_m}{dt} = -c_p \left(\frac{n_g}{1-n_g} \right) \left(\frac{\dot{m}}{A_g} \right) \frac{\partial T}{\partial x} - (\rho c_v + \beta P) \left(\frac{n_g}{1-n_g} \right) \frac{\partial T}{\partial t} + \beta T \left(\frac{n_g}{1-n_g} \right) \frac{\partial P}{\partial t}. \qquad (13)$$

We can average this equation over the entire regenerator and make a harmonic approximation for the time derivatives. The resulting approximation then becomes

$$\frac{\dot{Q}}{V_m} \approx c_p \left(\frac{n_g}{1-n_g} \right) \left(\frac{\dot{m}}{V_{rg}} \right) (T_h - T_c) - (\rho_0 c_v + \beta P_0) \left(\frac{n_g}{1-n_g} \right) i\omega T + \beta T_0 \left(\frac{n_g}{1-n_g} \right) i\omega P, \qquad (14)$$

where ω is the angular frequency, $i = \sqrt{-1}$, and $\dot{m}$, *T,* and *P* are time varying quantities that are relatively close to being in phase with each other in a well designed regenerative cryocooler. The negative sign in front of the second term on the right hand side of equation (14) indicates that changing temperature of the gas causes it to absorb some of the heat and act as its own regenerator material to some extent. For a good regenerator the gas temperature oscillation is relatively small and that term will be small compared to the other two terms. The last term is caused by the compression and expansion of the gas in the void space and is proportional to the frequency. The heat flow rate given in equation (14) leads to a temperature difference between the gas and the matrix, which can be minimized by increasing the surface area per unit volume (decreasing hydraulic diameter). The mass flow rate is independent of frequency for a given refrigeration power. Thus, the last term, which is proportional to frequency, becomes more important at high frequencies. The first term on the right hand side of equation (14) is minimized when decreasing the porosity and increasing the gas void volume. The other two terms are independent of the gas void volume, but are decreased when decreasing the porosity.

For first stage regenerators the temperature difference $T_h - T_c$ is quite large, but it is much smaller for second stage regenerators achieving 4 K with a hot end at 40 K. When typical parameters are used in equation (14) the first term is the order of 10^8 W/m^3 for a first stage regenerator and about 3x10^7 W/m^3 for a second stage regenerator. For a frequency of 2 Hz and a pressure amplitude of 0.5 MPa the last term is about 10^7 W/m^3 for a first stage regenerator of 0.65 porosity and about 4x10^6 W/m^3 for a second stage regenerator of 0.38 porosity. For a frequency of 20 Hz the last term becomes comparable to the first term for both the first and second stage regenerators, but the last term leads the first term by about 90°.

For 4 K regenerators the extra heat flow due to the last term can lead to a significant increase in the regenerator loss because of the limited matrix heat capacity for absorbing the heat transferred in a half cycle. The total heat transferred from the gas to the matrix in a half cycle is obtained by integrating equation (13) with respect to time and averaging the result over the regenerator length. The resulting equation becomes

$$\frac{\Delta Q}{V_m} \approx \rho_0 c_p \left(\frac{n_g}{1-n_g} \right) \left(\frac{1}{V_{rg} / V_E} \right) (T_h - T_c) - (\rho_0 c_v + \beta P_0) \left(\frac{n_g}{1-n_g} \right) 2T_1 + \beta T_0 \left(\frac{n_g}{1-n_g} \right) 2P_1, \qquad (15)$$

where V_E is the total swept volume of gas at the cold end, T_1 is the temperature amplitude, and P_1 is the pressure amplitude. For a 4 K regenerator with the hot end at 40 K the first term is on the order of 2x10^6 J/m^3 when $(V_{rg}/V_E) = 3$, and the last term is about 0.5x10^6 J/m^3. These values are independent of frequency unless (V_{rg}/V_E) varies with frequency.

A given value of (V_{rg}/V_E) determines the phase angle between the mass flows at the two ends of the regenerator. Ideally we wish to keep this phase angle independent of frequency, but in practice the phase angle and the volume ratio tend to increase with frequency because the optimum regenerator volume does not decrease with increasing frequency as fast as does the swept volume. A sufficient surface area, and, therefore, volume, is required to keep the gas temperature close to that of the matrix. We shall see in the next section that neat 4 K where the matrix heat capacity is very low the temperature gradient becomes very small to reduce the heat flow to the matrix, whereas at the warm end of the regenerator where the matrix heat capacity is much higher the gradient becomes much steeper than the average. A study of equations (13) - (15) provides qualitative understanding of regenerator behavior, but numerical procedures are needed for accurate quantitative results. In the next section we discuss such a numerical procedure but the equations are an aid to understanding the results.

NUMERICAL MODELS

The NIST numerical model known as REGEN3.2, which is based on finite difference equations for the conservation equations, was used for most of the calculations discussed here. This model is an upgrade of REGEN3.1 described previously [1,2]. There are no changes in the numerical procedure for solving the equations except for the addition of optional boundary conditions. In REGEN3.1 the mass flow amplitude and the phase angle at both ends of the regenerator are specified. The solution gives the average pressure and the pressure ratio. In REGEN3.2 we can specify the mass flow at the cold end and its phase with respect to the pressure at the cold end along with the desired pressure and pressure ratio and the program automatically finds the solution to meet these conditions. REGEN3.2 also has the ability to model layered regenerators and find the optimum transition location from one material to the other. This feature was used in the calculations discussed here. The materials specified for the layers were: $Er_{0.9}Yb_{0.1}Ni$, $ErAl_2$, $ErDy_{0.8}Ni_{0.2}$, $Er_{0.6}Pr_{0.4}$, and stainless steel.

In some cases we were not able to obtain convergence or the desire pressure and pressure ratio with REGEN3.2. These situations occurred with small regenerator volumes. We then used another NIST numerical model known as REGEN4. It is similar to REGEN3.2 except that the fixed boundary conditions are the mass flow at the cold end, the pressure at the hot end, and the phase angle between them. For the cases studied here with the cold end at 4 K there is a negligible difference between the pressure at the two ends, including the phase angle between them. However, this model converged in the regions that REGEN3.2 failed. We did compare the results from the two cases in a region where both models converged and found agreement in the calculated loss to with about 10%.

RESULTS OF CALCULATIONS

The results of calculations with REGEN3.2 and REGEN4 are put in dimensionless form to make the results of general use. The total regenerator loss consists of the ineffectiveness along with matrix conduction. For 4 K regenerators the conduction loss is usually negligible unless very low porosities can be used in an optimized manner. We then normalize the regenerator loss by the gross refrigeration power available at the cold end. For an ideal gas the gross refrigeration power is simply the isothermal PV power flow at the cold end, but with real gases it can be significantly different. Because conduction and pressure drop are usually very small in most 4 K regenerators, the aspect ratio is not very important. We then present our results in terms of the regenerator void volume, which is normalized by the swept volume of the gas at the cold end.

Figure 1 shows the results of calculated regenerator loss as a function of the normalized void volume. The cases studied are packed spheres with 38% porosity and parallel tubes with 15% and 38% porosity. Frequencies investigated were 2 Hz and 20 Hz, and the temperatures at the ends of the regenerators were 4 K and 40 K, unless indicated otherwise. The matrix material in all but one case was the optimized layered structure using Mix 1 discussed previously. The average pressure was 1.5 MPa and the pressure ratio was 2.0. The mass flow and pressure were in phase at the cold end. The results show that parallel tubes with a porosity of 0.15 have a significantly lower loss than the packed spheres with a porosity of 0.38. The single data point for parallel tubes with a porosity of 38% agrees with the packed spheres of 38% porosity. Thus, it appears that the porosity and not the geometry is the important parameter in this temperature range. This result is consistent with that suggested by equations (13) – (15).

The regenerator loss, as shown in Fig. 1, is higher for the higher frequency, but only at small regenerator void volumes. The surprising result is the fact that for all cases increasing void volume has little effect in decreasing the loss once a minimum volume has

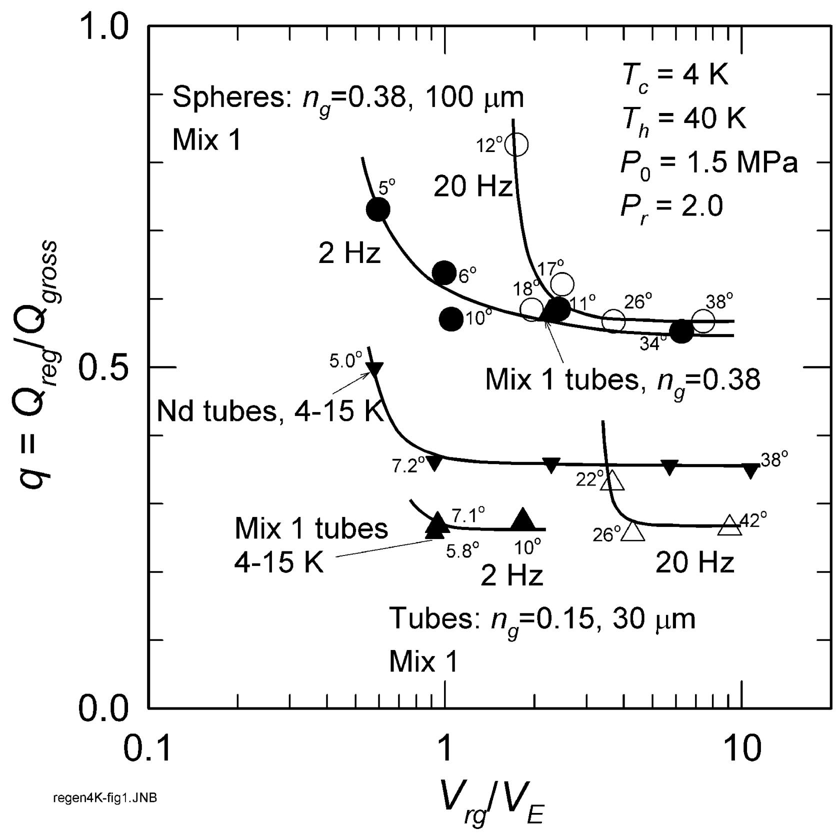

FIGURE 1. Reduced regenerator loss as a function of the ratio of regenerator void volume to expansion space swept volume. The phase angles between the mass flow at the two ends of the regenerator are shown for each data point.

been reached. The larger volumes are a disadvantage for two reasons. First the rare earth regenerator materials are very expensive. Second, the large volume leads to a large phase angle between the mass flows at the two ends of the regenerator. The large phase angle causes a large amplitude of mass flow rate at the warm end of the regenerator which becomes even larger when passing through the first stage. This large flow rate for a given PV power flow leads to large losses in the first stage regenerator and to large swept volumes for the compressor. We have found that a second stage phase angle greater than about 25° leads to significant increases in the losses of the first stage, unless we have the ability to significantly lag the flow at the cold end behind the pressure. That can be done with a displacer in Gifford-McMahon and Stirling refrigerators or with an inertance tube in large pulse tube refrigerators.

The flat curves above some minimum void volume in Fig. 1 indicate that the first term on the right hand side of equations (13) – (15) is no longer contributing to the loss at large void volumes. However a linear temperature profile in equation (13) would suggest that it should contribute a significant loss. Figure 2 shows the average temperature profile calculated from REGEN3.2 for the case of packed spheres with various normalized regenerator void volumes. Most of the temperature gradient occurs at the left (warm) end of the regenerator, especially with large void volumes. There is virtually no gradient through most of the regenerator in larger void volumes. The gradient is large at higher temperatures where the matrix heat capacity is large enough to absorb the heat from the gas. Near 4 K the matrix heat capacity is much smaller and the heat flow caused by gas flowing through a temperature gradient is forced to be small by the small temperature gradient. In this region heat flow is mainly due to the changing pressure in the last term of equation (13).

As the void volume is decreased the temperature profile begins to smooth out and become more linear. Below some minimum void volume the temperature profile “breaks” through at the cold end and transfers a large heat load to the cold end because of the large temperature swing. Figure 3 shows the temperature profile in the regenerator in this case of small void volume. The figure shows the average profile as well as the two limits of the motion of the profile in the two directions about the average. Note the “break” through at

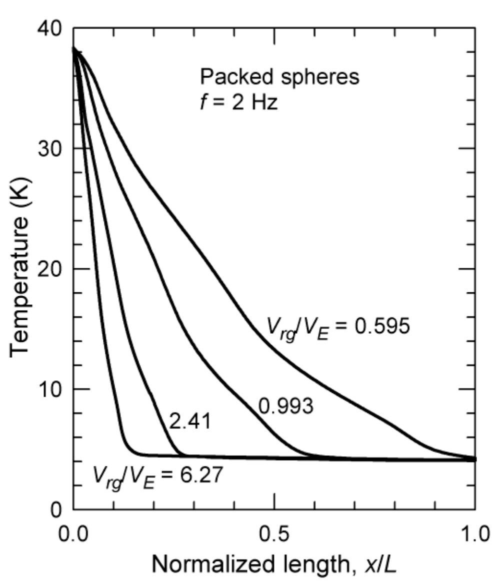

Figure 2. Calculated temperature profiles in packed sphere regenerators at 2 Hz for various regenerator gas volumes.

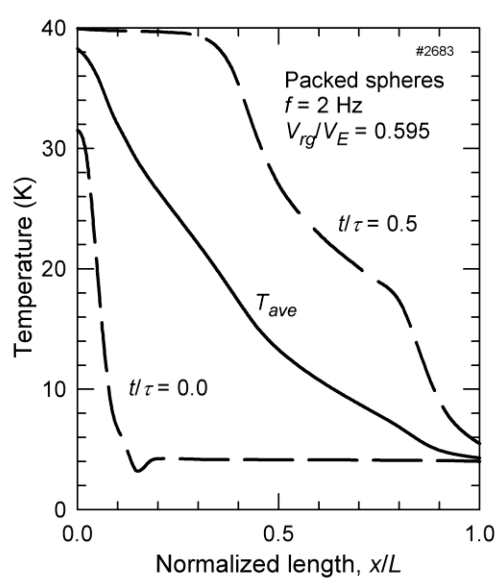

Figure 3. Calculated temperature profiles in a regenerator with small void volume. The average profile and instantaneous profiles at the two extremes are shown.

the cold end. This type of profile is in agreement with that seen by experiments [3,4] and with previous calculations [5,6].

The data point in Fig. 1 for the Mix 1 material with the hot end at 15 K shows very little reduction in the regenerator loss compared with the hot end at 40 K. Such a result is consistent with the fact that the temperature gradient is small at the cold end. The loss does increase significantly when the Mix 1 layered regenerator material is replaced with neodymium, which has a lower heat capacity.

The results presented here show that lower porosity can significantly reduce the regenerator loss for regenerators operating down to 4 K. The volume of the regenerator should be made small enough such that the temperature profile is spread out over most of the regenerator but that the profile does not "break" through at the cold end. As indicated in Fig. 1, the regenerator loss for a frequency as high as 20 Hz can be kept as low as that for 2 Hz, but only if the ratio of void volume to swept volume is made fairly large (>5). Such large volume ratios lead to large phase angles (>30°) between the mass flows at the two ends of the regenerator. With a sufficiently large phase shift at the cold end (mass flow lagging pressure) such a large phase angle could be accommodated in a two-stage system without significantly decreasing the performance of the first stage.

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