

THE SPECIFIC HEAT AT CONSTANT OSMOTIC PRESSURE OF He³ IN SUPERFLUID He⁴*

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The specific heat of He³ in superfluid He⁴; $C\mu_4$, has been measured for the case where μ_4 , the He⁴ chemical potential in solution, or the osmotic pressure are held constant. The measurements reported here cover the range 0.02 to 0.7 K and the He³ concentration range of 5 to 22%, and were made by the continuous flow calorimetric technique in a dilution refrigerator. Comparison with theory is made.

When He³ is dissolved in superfluid He⁴, then under equilibrium the condition $\nabla\mu_4 = 0$ must occur,¹ where μ_4 is the He⁴ chemical potential in solution. This chemical potential is easily shown² to be related to the osmotic pressure, Π , of He³ in He⁴ by

$$\Pi V_4 = (\mu_4 - \mu_4^0), \quad (1)$$

where V_4 is the partial molar volume of He⁴ in solution and μ_4^0 is the chemical potential of pure He⁴. For temperatures below about 0.5 K both V_4 and μ_4^0 are constant² and hence $\nabla\mu_4 = 0$ implies that $\nabla\Pi = 0$ at these temperatures. In an experimental situation where the superfluid He³-He⁴ solution is allowed to flow with little impedance for the non-superfluid He³ component, the flow takes place at constant Π , or more exactly, constant μ_4 . Such a process occurs on the dilute side of a dilution refrigerator where the He³ actually diffuses through the nearly stationary He⁴. The increment of power, $\Delta\dot{Q}$ absorbed in raising the temperature of the stream by ΔT is simply related to the specific heat of He³ at constant μ_4 by the expression

$$C\mu_4 = \Delta\dot{Q}/(\dot{n}\Delta T), \quad (2)$$

where \dot{n}_3 is the He³ molar flow rate.

Shown in Fig. 1 are theoretical calculations previously made² for $C\mu_4$ for $0\text{K} < T < 1\text{K}$ which apply to the dilution refrigerator process where the low temperature end of the dilute stream is in equilibrium with pure He³. These calculations were done using the weakly interacting Fermi gas model³ for the He³ in He⁴. Values for $C\mu_4$ are used for calculations on heat exchangers for dilution refrigerators. Until now no experimental test has been made of the calculated values. Such experimental results in the approximate range $0.1\text{K} < T < 0.5\text{K}$ can also lead to information about the interaction potential of He³ in He⁴, which has a reasonably strong influence on $C\mu_4$ in such a temperature range. The specific heat $C\mu_4$ is thermodynamically related to the more familiar specific heat at constant

volume, C_v , or concentration by the expression⁴

$$C\mu_4 = C_v + T(\partial\Pi/\partial T)_v (\partial v/\partial T)\Pi, \quad (3)$$

where v is the volume per mole of He³ in solution. For $T < 0.04\text{K}$ equation (3) becomes²

$$C\mu_4 = (107.2 + 6.1 \times 10^3 T^2)T\text{J/mol K}, \quad (3')$$

provided the limiting solubility, X_0 , of He³ in He⁴ at $T = 0\text{K}$ is 0.0640.⁵ For $X_0 = 0.0684$ ⁶ the expression is reduced by 4.1%. In fact accurate measurements of $C\mu_4$ for $T < 0.04\text{K}$ can be used to determine X_0 . Additional and more accurate work is in progress to make such a determination of X_0 . The deviations shown in Fig. 1 from the classical 5R/2 value at higher temperatures are from two sources. The upward curvature at the beginning of the higher temperature curves is a result of the interaction potential of He³ in He⁴. The upward curvature of the high temperature end occurs because $\nabla\Pi = 0$ no longer holds.

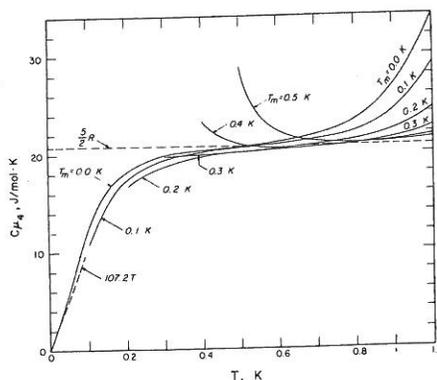


Fig. 1. Calculated Specific Heat of He³ at Constant μ_4 as a Function of Temperature for Various Mixer Temperatures.

The experimental arrangement for making the measurements of μ_4 with a dilution refrigerator is shown in Fig. 2. The mixer heater and the incoming concentrated He³ are on the outside of a spiral of copper foil,

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which has one side coated with sintered copper powder. The spiral is composed of 20 strips, insulated from each other by a polyester film to prevent eddy current heating from the field of the mutual inductance coil used to measure the susceptibility of the CMN powder. In this experiment the spiral serves as a baffle to prevent any incoming warm liquid from heating the returning dilute liquid above the temperature of the interface. Heavy copper wires hard soldered to each strip of the spiral and passing through the mixer walls provide a good heat sink for other experiments. The dilute stream passes through about 10 cm of 3 mm o.d. \times 0.1 mm wall CuNi tube before entering cell A. This large tube is used to prevent viscous heating at temperatures as low as 15 mK. Both cell A and B are lined with various types of sintered copper powder to promote heat transfer (and so that measurements of Kapitza resistance to these sintered materials can be measured simultaneously). Both cells have a heater and resistance thermometer glued to the outside, but only the heater on cell A and the thermometer on cell B are used for the measurements of $C\mu_4$. The mixer temperature, T_m , can be held constant by the mixer heater. The resistance thermometers are calibrated against the CMN thermometer when no heat is applied to cell A. During the j^{th} run an amount of power \dot{Q}_j is fed to the heater on cell A and the resultant stream temperature, T_j , is measured at cell B about 5 cm downstream. As the power is increased with each run, the mixer heating must be steadily reduced to maintain the same T_m . The values $\Delta\dot{Q} = \dot{Q}_j - \dot{Q}_{j-1}$ and $\Delta T = T_j - T_{j-1}$ are used in equation (2) to find $C\mu_4$, with appropriate corrections for slight variations in mixer temperature and flow rate. Both cell A and B are situated below the mixer so that the more dense warmer liquid will not cause convective instabilities in the large tube between the cells and the mixer.

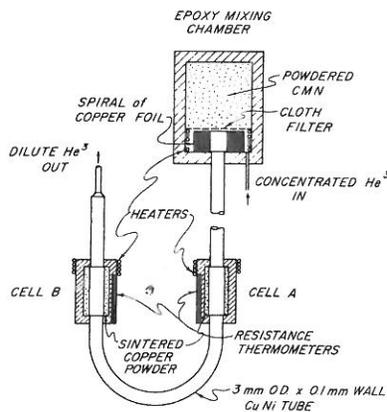


Fig. 2. Schematic Diagram of Apparatus to Measure $C\mu_4$.

The He^3 flow rate \dot{n}_3 is obtained from the expression $\dot{n}_3 = \dot{n}X_v$, where \dot{n} is the total molar flow rate and X_v is the He^3 concentration in the circulated gas. The concentration X_v is measured by a mass spectrometer leak detector calibrated against known concentrations of He^3 in He^4 . The total flow \dot{n} is measured at room temperature at the outlet of the circulating pump. Readings of the absolute pressure, P , and the pressure drop, ΔP , across a small impedance in the gas line are taken. The flow rate is then $\dot{n} = kP\Delta P$, where the constant k is determined to within 1% from He^3 - He^4 flow measurements into a calibrated volume. The time lag for a change in \dot{n}_3 in the dilute stream to be sensed in the gas at room temperature may be several minutes, but \dot{n} could be held constant to within 0.5% for a period of hours.

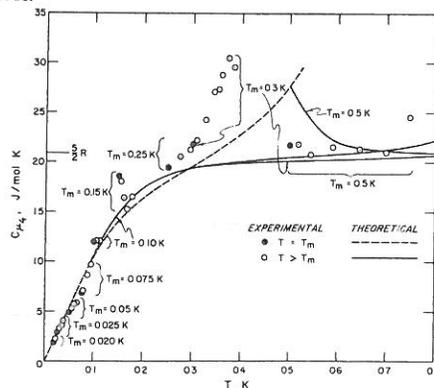


Fig. 3. Comparison of Experimental and Theoretical Results for $C\mu_4$.

Results of the measurements to date are shown in Fig. 3. An upper limit for the inaccuracy of these data is estimated to be about 7%, which contains data scatter of about 5%. Efforts are being made to analyze and reduce the uncertainty, most of which is from measurements of ΔT . The data include measurements taken with several mixer temperatures. The filled circles represent the data extrapolated back to $T = T_m$ and the dashed line is the predicted theoretical behavior for $T = T_m$. The few solid lines give representative behavior of the theoretical $C\mu_4$ for $T > T_m$ and are to be compared with the unfilled circles. Agreement between theory and experiment is very good at the low temperatures, whereas at temperatures between 0.2 K and 0.4 K, there is considerable disagreement. The disagreement would suggest that the weakly interacting model for He^3 in He^4 is breaking down at these higher temperatures and concentrations. For $T_m = 0.3$ K the He^3 concentration is about 12% and increases to 22% at $T_m = 0.5$ K. Further work is in process to extend the measurements to where T is at least twice that of T_m .

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References

- ¹ J.C. Wheatley, *Am. J. Phys.*, **36** (1968) 181.
- ² R. Radebaugh, National Bureau of Standards Technical Note 362 (1967).
- ³ J. Bardeen, G. Baym, and D. Pines, *Phys. Rev.*, **156** (1967) 207.
- ⁴ B.S. Neganov, Preprint, Joint Institute for Nuclear Research, Dubna (USSR), (JINR-P13-4014), (1968). [English transl. BNL-TR-234].
- ⁵ D.O. Edwards, E.M. Ifft, and R.E. Sarwinski, *Phys. Rev.*, **177** (1969) 380.
- ⁶ R.I. Schermer, L. Passel, and D.C. Rorer, *Phys. Rev.*, **173** (1968) 277.